CORRECTION TO "SPACES OF RIEMANN SURFACES AS BOUNDED DOMAINS"[1]

BY LIPMAN BERS

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In [3] I sketched a proof of the theorem: *Every Teichmüller space* $T_{g,n}$ *is a bounded domain in complex number space.*

This proof is invalid since Lemma B is false. The error in the argument occurs on page 101, lines 8–12. The theorem is nonetheless true. A complete proof will appear elsewhere; a brief outline follows. The same proof was found, simultaneously and independently, by Lars V. Ahlfors.

Let G be a Fuchsian group without elliptic elements and with the unit circle as a limit circle. Denote by U the unit disc and by V the domain $1<|z|\leqq\infty$. The Riemann surfaces $S=V/G$ and $\bar{S}=U/G$ are mirror images of each other.

Let M be the set of complex-valued measurable functions $\mu(z)$ such that $|\mu(z)|\leqq k(\mu)<1$, $\mu\equiv 0$ in U, and $\mu(z)d\bar{z}/dz$ is invariant under G. For $\mu\in M$ let $z\rightarrow w^\mu(z)$ be the homeomorphism of the plane onto itself which satisfies the Beltrami equation $w_{\bar{z}}=\mu w_z$ and is normalized by the conditions $w^\mu(0)=0$, $w^\mu(1)=1$. Then $G^\mu=w^\mu G(w^\mu)^{-1}$ is a discontinuous group of Möbius transformations and $S^\mu=w^\mu(V)/G^\mu$ a Riemann surface. Also, w^μ defines a quasiconformal mapping f^μ of S onto S^μ and thus a point in the Teichmüller space $T(S)$; all points in this space can be so obtained. We say that μ and ν are equivalent if they define the same point in $T(S)$, i.e. if S^μ is conformal to S^ν and f^μ homotopic to f^ν. This is so if and only if there is a Möbius transformation C such that $C(w^\mu(z))=w^\nu(z)$ in U (cf. [2]).

Holomorphic quadratic differentials on $\bar{S}$ may be represented by G$-$automorphic forms of weight (-4) in U, i.e. by holomorphic functions $\phi(z)$, $z\in U$, with $\phi(z)dz^2$ invariant under G. We define the norm $\|\phi\|$ to be the supremum of $\lambda|\phi|$ where $\lambda(z)=(1-|z|^2)^2$. The quadratic differentials of finite norm form a complex Banach space B.

For $\mu\in M$ the function w^μ is holomorphic in U and so is its Schwarz derivative ϕ^μ; note that ϕ^μ depends only on the equivalence class $[\mu]$ of μ. One verifies directly that $\phi^\mu(z)dz^2$ is G-invariant, and by a theorem of Nehari [4] we have that $\|\phi^\mu\|\leqq 6$. Knowing ϕ^μ we may recon-

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