RESEARCH PROBLEMS

4. Richard Bellman: *Theory of numbers*

The question of the solubility in rational integers of the congruence $x^2+ax+b\equiv 0(p)$ can be decided by examining a polynomial congruence $a^2-4b\equiv y^2(p)$. Similarly, one can study the solubility of $x^3+ax+b\equiv 0(p)$ and $x^4+ax+b\equiv 0(p)$.

It is conjectured that the solubility in rational integers of $x^5+ax+b\equiv 0(p)$ cannot be discussed in terms of any finite system of polynomial congruences involving the coefficients a and b, where the number of congruences and the degrees are independent of the prime p. (Received May 1, 1961.)

5. Richard Bellman: *Control processes*

Consider the problem of minimizing the quadratic functional

$$J(y) = \int_0^T [(x, Ax) + 2(x, By) + (y, Cy))]dt,$$

over all vector functions y related to x by means of the linear differential equation $dx/dt=Ax+y$, $x(0)=c$, and subject to the component constraints $|y_i|\leqq m_i$, $i=1, 2, \cdots, N$. Can one obtain an explicit analytic solution? (Received May 1, 1961.)

6. Herbert S. Wilf: *Reciprocal bases for the integers*

It is well known that every integer is the sum of reciprocals of distinct integers. Let us call a sequence S: $\{n_1, n_2, n_3, \cdots\}$ of distinct integers an R-basis if every integer is the sum of reciprocals of finitely many integers of S. It is clearly necessary that

$$\sum n_j^{-1} = \infty$$

though this is not sufficient, as can be seen by considering the primes. Yet it is not necessary to use all the integers, since $a, 2a, 3a, \cdots$ will obviously do, for any a.

Are the odd numbers an R-basis? Is every arithmetic progression an R-basis? Does an R-basis necessarily have a positive density? lower density? If S contains all integers and $f(n)$ is the least number required to represent n, what, in some average sense, is the growth of $f(n)$? (Received June 12, 1961.)