A DECOMPOSITION THEORY FOR UNITARY REPRESENTATIONS OF LOCALLY COMPACT GROUPS

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The purpose of this note is to announce a decomposition theory for unitary representations over a separable complex Hilbert space, prototyped after the type I theory given in [6], but which applies to all separable locally compact groups. The theory uses the central (or canonical) decomposition throughout and the "building blocks" are the primary (or factor) representations. (A representation is *primary* if it cannot be expressed as the direct sum of two disjoint representations. Two representations L and M are *disjoint*, denoted $L \mathbin{\delta} M$, if no subrepresentation of one is equivalent to any subrepresentation of the other.)

PROPOSITION 1. *Let L denote a representation of a separable locally compact group G with a decomposition $L \simeq \int_Z L^y d\mu(y)$ over a separable Borel space Z, such that the range of the corresponding projection-valued measure is contained in the center of the commuting algebra, $\mathcal{R}(L, L)$, of L. Then there exists a Borel subset Z' of Z such that $\mu(Z-Z')=0$ and $L^y \mathbin{\delta} L^{y'}$ whenever $y, y' \in Z'$ and $y \neq y'$.*

This result has also been announced by Naimark [9, Theorem 1] in the important special case where the decomposition is the central decomposition. The proof of Proposition 1 is a simple perturbation of a proof given by Guichardet [3] for the special case of the central decomposition of a multiplicity free representation.

A representation L *covers* a representation M, denoted $L \} M$, if no subrepresentation of M is disjoint from L. L and M are said to be *quasi-equivalent*, denoted $L \sim M$, if L covers M and M covers L.

PROPOSITION 2. *Two representations L and M are quasi-equivalent if and only if they have central decompositions over the same measure space, say $L \simeq \int_Z L^y d\mu(y)$ and $M \simeq \int_Z M^y d\mu(y)$, such that $L^y \sim M^y$ for μ-almost all y.*

The proof of Proposition 2 is an adaptation of known facts about direct integrals of isomorphisms of von Neumann algebras. (Cf. [1].)

The collection Q of all quasi-equivalence classes of representations of a group G (always assumed to be separable locally compact), partially ordered by the covering relation defined above, is a distribu-

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