STABLE EQUIVALENCE OF DIFFERENTIABLE MANIFOLDS

BY BARRY MAZUR

Communicated by Edwin Moise, March 1, 1961

A natural question, of great generality, various special forms of which are often asked in differential topology, is the following:

Let M_1, M_2 be differentiable n-manifolds, ϕ: $M_1 \to M_2$ a continuous map which is a homotopy equivalence between M_1 and M_2. When is there a differentiable isomorphism

$$\Phi: M_1 \to M_2$$

in the same homotopy class as ϕ?

For example, there is the Poincaré Conjecture which poses the question when M_1 is an n-sphere (see Smale [2], Stallings [3]).

I should like to suggest a certain simpleminded "stabilization" of the above question.

I shall say that Φ is a k-equivalence between M_1 and M_2, denoted:

$$M_1 \underset{\underset{(k)}{\approx}}{\overset{\Phi}{\to}} M_2$$

for k a non-negative integer, if Φ is a differentiable isomorphism between $M_1 \times R^k$ and $M_2 \times R^k$,

$$\Phi: M_1 \times R^k \underset{\approx}{\to} M_2 \times R^k.$$

Now our original question may be reformulated as follows:

(P_k) If ϕ: $M_1 \to M_2$ is a homotopy equivalence, when is there a k-equivalence

$$M_1 \underset{\underset{(k)}{\approx}}{\overset{\Phi}{\to}} M_2$$

in the same homotopy class as ϕ? (I.e., such that

$$\begin{array}{ccc} M_1 \times R^k & \overset{\Phi}{\to} & M_2 \times R^k \\ \downarrow & & \downarrow \\ M_1 & \xrightarrow{\quad\phi\quad} & M_2 \end{array}$$

is homotopy commutative.)