THE KRULL-SCHMIDT THEOREM FOR INTEGRAL GROUP REPRESENTATIONS

BY IRVING REINER[1]

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Let R_0 be the ring of algebraic integers in an algebraic number field K, let P be a prime ideal in R_0, and let R_P (or briefly R) denote the ring of P-integral elements of K. Choose $\pi \in R_0$ such that πR is the unique maximal ideal in R. Further let K^* be the P-adic completion of K, with ring of P-adic integers R^*. For a fixed finite group G, we understand by the term "R_0G-module" a left R_0G-module which as R_0-module is torsion-free and finitely-generated; analogous definitions hold for RG- and R^*G-modules.

Swan [**9**; **10**] has recently proved that the Krull-Schmidt theorem is valid for projective R^*G-modules. We show here the following main result, which is a consequence of some work of Maranda [**3**; **4**]:

THEOREM 1. *The Krull-Schmidt theorem holds for arbitrary R^*G-modules, that is, if $M_1, \cdots, M_r, N_1, \cdots, N_s$ are indecomposable R^*G-modules such that*

$$M_1 \dotplus \cdots \dotplus M_r \cong N_1 \dotplus \cdots \dotplus N_s \tag{1}$$

(the notation indicating external direct sums), then $r=s$, and after renumbering the $\{N_j\}$ if need be, $M_1 \cong N_1, \cdots, M_r \cong N_r$.

To prove this and some corollaries we make use of the following results of Maranda [**3**; **4**].

(i) Let M and N be R^*G-modules, and let e be the largest integer for which P^e divides the order of G. If $M \cong N$ then

$$M/\pi^d M \cong N/\pi^d N \qquad \text{as } (R^*/\pi^d R^*)G\text{-modules} \tag{2}$$

for all d.

Conversely if (2) holds for some $d>e$, then $M \cong N$. Furthermore, the same result holds for RG-modules.

(ii) Let M and N be RG-modules. Then $M \cong N$ if and only if $R^*M \cong R^*N$.

(iii) Let M be an R^*G-module. If M is decomposable, so is $M/\pi^d M$ for all d. Conversely if $M/\pi^d M$ is decomposable as $(R^*/\pi^d R^*)G$-module for some $d>2e$, then M is also decomposable.

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