# RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

## ONTO INNER DERIVATIONS IN DIVISION RINGS

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1. **Introduction.** Kaplansky [3] proposed the following problem: Does there exist a division ring $\Delta$ each element of which is a sum of additive commutators $ab-ba$? In [1] Harris gave a strongly affirmative solution to this problem by constructing division rings $\Delta$ in which each element $c=ab-ba$ for some $a, b\in\Delta$. Recently Meisters [4] has studied rings $R\neq(0)$ in which for any triple of elements $a, b, c\in R$ with $a\neq b$ there exist solutions of the equation $ax-xb=c$. He has shown that (1) $R$ is a division ring in which every noncentral element induces an onto inner derivation and (2) if $R$ is separable algebraic over its center, then $R$ is commutative. Actually one can prove the more general result that in a division ring $R$ of the preceding type all algebraic elements (over the center) are central. (Hence if $R$ is noncommutative, each noncentral element $t\in R$ is transcendental over the center of $R$ and induces an onto inner derivation.)

In view of the above work it seems natural to investigate the question of existence of division rings possessing onto inner derivations. We give a partial answer to this question which implies (in some heuristic sense) that Harris' examples (at least for char. $p>0$) are normative rather than pathological. More precisely we sketch a proof of the following theorem: For each division ring $\Delta$ of char. $p>0$ one can construct an extension division ring $E$ with the property that there exists an element $t\in E$ (lying in the centralizer of $\Delta$) whose associated inner derivation $D_t$ is an onto map: $D_t(E)=E$.

2. **Preliminaries.** We shall make consistent use of the following facts: (1) Any noncommutative ring $R$ with an identity having the common right multiple property has a right quotient ring $Q(R)$, i.e., every element of $Q(R)$ has the form $ab^{-1}$, $a$, $b\in R$, $b$ regular, and all regular elements of $R$ are invertible in $Q(R)$. (2) If $\Delta$ is a division ring and $D$ a derivation of $\Delta$ into itself, then $\Delta[x; D]$, the ring of differential polynomials over $\Delta$ in the indeterminate $x$, has the com-