AREA OF DISCONTINUOUS SURFACES

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1. A general theory of surface area, [**1**; **2**], exists for the non-parametric case. Thus, area is defined for all measurable f on the unit square $Q=I\times J$. The area functional is lower semi-continuous with respect to almost everywhere convergence and agrees with the Lebesgue area for continuous f. On the other hand, for continuous parametric mappings T of the closed unit square Q into euclidean 3-space E_3, Lebesgue area is not lower semi-continuous with respect to almost everywhere convergence nor even, as C. J. Neugebauer has shown, [**3**], with respect to pointwise convergence.

It thus appears that a theory of parametric surface area must be restricted to surfaces which cannot deviate too far from the ones given by continuous mappings. In this paper, we develop the beginnings of a theory for a class of surfaces which we call linearly continuous.

2. Let f be a real function defined on Q and, for every u, let f_u be defined by $f_u(v)=f(u, v)$ and let f_v be defined similarly. Then f is linearly continuous if f_u is continuous for almost all u and f_v is continuous for almost all v. A mapping $T: x=x(u, v),\ y=y(u, v),\ z=z(u, v)$ of Q into E_3 is linearly continuous if x, y, z are linearly continuous.

A sequence $\{f_n\}$ of functions converges linearly to a function f if $(f_n)_u$ converges uniformly to f_u for almost all u, and $(f_n)_v$ converges uniformly to f_v for almost all v. A sequence $T_n: x=x_n(u, v), y=y_n(u, v), z=z_n(u, v)$ converges linearly to a mapping $T: x=x(u, v), y=y(u, v), z=z(u, v)$ if $\{x_n\}, \{y_n\}, \{z_n\}$ converge linearly to x, y, z, respectively.

Let P be the set of quasi linear mappings from Q into E_3. For $p, q\in Q$ let

$$\begin{aligned} d(p, q) = \inf[k&: \text{there are sets } A_k \subset I,\ B_k \subset J,\\ &m(A_k) > 1 - k,\ m(B_k) > 1 - k, \text{ and } |p(u, v) - q(u, v)| < k\\ &\text{on } (A_k \times J) \cup (I \times B_k)]. \end{aligned}$$

It is easy to verify that P is a metric space and that $\{p_n\}$ converges to p in this space if and only if it converges linearly. Let E be the elementary area functional on P. It is not hard to prove

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