RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

A NEW CLASS OF PROBABILITY LIMIT THEOREMS

BY JOHN LAMPERTI[1]

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Suppose that $\{X_n\}$ is a Markov process with states on the non-negative real axis and stationary transition probabilities. Define

$$\mu_k(x) = E[(X_{n+1} - X_n)^k \mid X_n = x], \qquad k = 1, 2, \cdots; \tag{1}$$

we assume that for each k, $\mu_k(x)$ is a bounded function of x. Assume also

$$\lim_{x\to\infty} \mu_2(x) = \beta > 0, \qquad \lim_{x\to\infty} x\mu_1(x) = \alpha > -\frac{\beta}{2}\cdot \tag{2}$$

We shall say that the process $\{X_n\}$ is *null* provided that

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \Pr(X_i \leqq M) = 0 \tag{3}$$

for all finite M. A class of examples satisfying all the conditions imposed so far is afforded by Markov chains on the integers with transition probabilities of the form

$$p_{j,j+1} = \frac{1}{2}\left[1 + \frac{\alpha}{j} + o\left(\frac{1}{j}\right)\right] > 0,\ p_{j,j-1} = 1 - p_{j,j+1} \quad \text{if } j \neq 0; \tag{4}$$

$$p_{01} = 1 - p_{00} > 0.$$

For such chains (random walks) the *null* condition is known to hold if $\alpha > -1/2$ $(= -\beta/2)$. In many (but, so far at least, not all) other cases, it can be shown that (3) follows automatically from the other hypotheses.

For a process $\{X_n\}$ satisfying the above assumptions, there is an analogue of the central-limit theorem:

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