A CHARACTERIZATION OF DISCRETE SOLVABLE MATRIX GROUPS

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Introduction. In [1], we introduced a class of solvable groups which we called algebraic strongly torsion free S groups. We will show in this note how these groups can be modified and used to characterize those solvable groups which can be imbedded as discrete subgroups of the group $GL(n, C)$ for some n.

1. **Preliminary discussion and definitions.** Let Γ be a strongly torsion free S group in the sense of H. C. Wang [5]; i.e., Γ satisfies the diagram

$$1 \to D \to \Gamma \to Z^S \to 1$$

where D is a finitely generated torsion free nilpotent group and where Z^S is the additive group of integers taken s times. It is shown in [1] that there exists a unique maximal nilpotent subgroup M of Γ which contains the commutator subgroup $[\Gamma, \Gamma]$. Clearly M is a characteristic subgroup of Γ and torsion free. For any finitely generated torsion free nilpotent group G, we will use $N(G)$ to denote the unique connected simply connected nilpotent Lie group which contains G as a discrete uniform subgroup. We will use $N_C(G)$ to denote the complexification of this Lie group. With this convention made, we may now let $A_1(\Gamma)$ denote the image of Γ in the automorphism group of $N_C(M)$, $A(N_C(M))$, obtained by forming inner automorphisms of Γ. Let $\Gamma^* \subset \Gamma$ be a characteristic subroup of Γ such that Γ/Γ^* is finite, $\Gamma^* \supset M$ and Γ^*/M is torsion free. We may apply the construction of H. C. Wang [5] to the group $S = \Gamma^* N_C(M)$ and obtain $S \subset F \cdot T$, where F is the maximal unipotent subgroup, $F \supset N_C(M)$ as a characteristic subgroup, T is abelian and the dot denotes semi-direct products. We may form $A_1(F) \subset A(N_C(M))$.

Definition. We will say that a strongly torsion free S group is complex algebraic if there exists an abelian analytic group of semi-simple elements T^* in $A(N_C(M))$ such that

1. T^* is in the normalizer of $A_1(F)$,
2. $A_1(\Gamma) \subset A_1(F) \cdot T^*$

where the dot denotes the semi-direct product.

Remark 1. T^* can be considered as an abelian analytic semi-simple group of automorphisms of $N_1(F)$, where $N_1(F) \supset F$, $N_1(F)$ is connected simply connected nilpotent Lie group and $N_1(F)/F$ is compact.

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