TWO ELEMENT GENERATION OF THE SYMPLECTIC GROUP[1]

P. F. G. STANEK

Communicated by Irving Kaplansky, November 7, 1960

Albert and Thompson [1] have given two generators for the projective unimodular group over any finite field, one of which has period (group order) two. Using the same method, it is possible to prove a similar result for the symplectic group, which may be described as the group of linear transformations on an even-dimensional vector space which leave invariant a skew-symmetric bilinear form.

Let

$$H = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

be a $2n$ by $2n$ matrix and let $G(2n, q)$ be the group of all matrices X, with entries from $GF(q)$, which satisfy $XHX^T = H$, where X^T is the transpose of X. Denote by $S(2n, q)$ the group $G(2n, q)$ modulo its center. $S(2n, q)$ is known to be simple, except for $n=1$, $q=2$ or 3, and $n=2$, $q=2$.

The following three types of matrices are known to be generators of $G(2n, q)$ (see [2]):

(i) translations:

$$T = \begin{pmatrix} I & S \\ 0 & I \end{pmatrix}, \quad \text{where} \quad S^T = S;$$

(ii) rotations:

$$R = \begin{pmatrix} U & 0 \\ 0 & U^{T^{-1}} \end{pmatrix}, \quad \text{where} \quad \det U \neq 0;$$

(iii) semi-involutions:

$$S = \begin{pmatrix} Q & I - Q \\ Q - I & Q \end{pmatrix},$$

where Q is a diagonal matrix of 0's and 1's, so that $Q^2 = Q$, $(I-Q)^2 = I-Q$.

Denote by E_{ij} the n by n matrix with a 1 in the ijth entry and zeros elsewhere. With α primitive in $GF(q)$, set

[1] This research was supported in part under NSF G-9504.