HOLOMORPHIC DIFFERENTIALS AS FUNCTIONS OF MODULI[1]

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The purpose of this note is to strengthen the results of [3] and to indicate a very brief derivation of some theorems announced without proof in [1; 3].

We begin by indicating a correction to [3]. Let S_1 and S_2 be Riemann surfaces, f an orientation preserving (orientation reversing) homeomorphism of bounded eccentricity of S_1 onto S_2 and $[f]$ the homotopy class of f; then $(S_1, [f], S_2)$ is called an even (odd) coupled pair of Riemann surfaces. The definition of equivalence of such pairs given in [3] is imprecise and garbled by misprints. The correct definition reads: $(S_1, [f], S_2)$ and $(S_1', [f'], S_2')$ are called *equivalent* if there exist conformal homeomorphisms h_1 and h_2 with $h_1(S_1)=S_1'$, $h_2(S_2)=S_2'$ and $[h_2 f]=[f' h_1]$; the two pairs are called *strongly equivalent* if $S_2'=S_2$ and there exists a conformal homeomorphism h with $h(S_1)=S_1'$ and $[f]=[f'h]$. If S_0 is a Riemann surface, then the Teichmüller space $T(S_0)$ can be thought of as the set of strong equivalence of even pairs $(S, [f], S_0)$ (and not of simple equivalence classes as stated in [3]).[2]

From now on we assume that S_0 is a fixed closed Riemann surface of genus $g>1$, and we write T instead of $T(S_0)$. T has a natural complex analytic structure and can be represented as a bounded domain, homeomorphic to a ball, in complex number space with coordinates (moduli) $\tau_1, \cdots, \tau_{3g-3}$ (cf. [1; 2]). Points of T will be denoted by τ. We may assume that S_0 is given as the unit disc modulo a fixed-point-free Fuchsian group, and that $\tau=0$ corresponds to the pair $(S_0, [\text{identity}], S_0)$.

THEOREM I. *One can associate to every $\tau \in T$ a bounded Jordan domain $D(\tau)$ and $2g$ Möbius transformations $z \to A_j(z, \tau)$, $z \to B_j(z, \tau)$, $j=1, \cdots, g$, such that the following conditions are satisfied.*

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[2] We also note the following errata to [2; 3]. On p. 94, l. 19, replace (ζ) by $\mu(\zeta)$. On p. 96, l. 15, replace the subscript j by $2j$. On p. 97, l. 21, replace C_r by C^r. On p. 100, l. 4, replace 'covering' by 'covering space.' On p. 103, equation (9) replace the exponent $3g-3n+n$ by $3g-3+n$.