ON GLOBAL SOLUTIONS FOR PARTIAL DIFFERENTIAL EQUATIONS OF FIRST ORDER[1]

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In this note we state a theorem which guarantees the existence and uniqueness of a global solution for the Cauchy initial value problem for a complete, regular system of partial differential equations of first order for one unknown function on a manifold—all data being of class C^∞.

This result depends on one hand on an investigation of the following problem for a m-dimensional C^∞-manifold W, which is foliated by r-dimensional leaves, the leaf passing through $w \in W$ being denoted by $C_{(w)}$: Given a k-dim. submanifold A of W with $k+r \leqq m$, find a foliated C^∞-manifold S and a C^∞-immersion $j: S \to W$ such that (i) $j(S) = \bigcup_{a \in A} C_{(a)}$, (ii) each leaf of S is mapped under j onto a leaf of W. Concerning this problem we prove: If for all $a \in A$ the tangent spaces of A and $C_{(a)}$ at the point a have only the zero vector in common and if—in case $r > 1$—for all $a \in A$ the leaf $C_{(a)}$ is simply connected, then there exist S and j with the desired properties. From this result and from the theory of characteristics for the above mentioned systems (essentially due to Cauchy, Lie, Goursat and E. Cartan [2])[2] we obtain the announced theorem. Complete proofs and other applications will be published elsewhere.

NOTATIONS. "Manifold" will always mean a connected C^∞-manifold which is Hausdorff and has a countable basis of open sets. If N is a manifold, N_n denotes the tangent space to N at $n \in N$, $\mathfrak{F}(N)$ the ring of global C^∞-functions on N, $\mathfrak{X}(N)$ the $\mathfrak{F}(N)$-module of global C^∞-vector-fields on N, and a manifold L is called a submanifold of N, if the underlying set of L is a subset of the underlying set of N and if the inclusion map $i: L \to N$ is a C^∞-immersion.

DATA AND DEFINITIONS. Let M be a d-dim. manifold and $T^*(M)$ its cotangent-bundle. Consider the $(2d+1)$-dim. product-manifold $T^*(M) \times R$, where R denotes the real line. Let W be a *system of r partial differential equations of first order for one unknown function on M*, i.e., W is a submanifold of $T^*(M) \times R$ of codimension r. A *local*

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[2] In trying to understand E. Cartan's description of the theory of characteristics and in developing a more intrinsic form of it for our special case (sketched below) a paper of M. Breuer [1] has been of help to me.