RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

A PROOF OF THE POWER SERIES EXPANSION WITHOUT CAUCHY'S FORMULA

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In this note a proof will be outlined that a differentiable function of a complex variable has a power series expansion. This work is independent of any integration theory and in particular is independent of Cauchy's formula. These results depend upon one of the basic theorems of topological analysis—namely that if f is differentiable and nonconstant in a region R of the complex plane, then f is an open map, i.e., if O is open in R, $f(O)$ is open in E_2 [5, p. 76].

We adopt the convention that if $f(z)$ is differentiable, the symbol $(f(z)-f(z_0))/(z-z_0)$ represents the function $h(z)=(f(z)-f(z_0))/(z-z_0)$ when $z\neq z_0$ and $h(z)=f'(z_0)$ when $z=z_0$. The function h will be continuous at z_0. A region R is a connected open subset of the complex plane and all functions are from subsets of the complex plane into the complex plane.

LEMMA 1. *Suppose O is a bounded open set and f is continuous on $\overline{O}$ and open on O. Then if W is a complementary domain (component of the complement) of $f(\overline{O}-O)$, $f(O)\cap W\neq\emptyset$ implies $f(O)\supset W$.*

PROOF. Suppose $f(O)\cap W\neq\emptyset$. Then $f(O)\cap W$ is open in W and $f(O)\cap W=f(\overline{O})\cap W$ is closed in W. Since W is connected, $f(O)\supset W$.

LEMMA 2. *Suppose V is open and $p\in V$. If f is continuous on V and open on $V-p$, then f is open on V.*

PROOF. Suppose D is an open set of V containing p. Show that $f(p)$ is in the interior of $f(D)$. Let S be a circle with center p and $S\cup I(S)\subset D$. If $f(p)\in f(S)$ then $f(p)$ will be in the interior of $f(D)$. Assume $f(p)\notin f(S)$ and apply Lemma 1. Let $O=I(S)-p$ and T be the interior of a circle conaining $f(p)$ with $T\cap f(S)=\emptyset$. Note $(T-f(p))\cap f(O)\neq\emptyset$ and that if W is the complementary domain of $f(\overline{O}-O)$ containing $T-f(p)$, $f(O)\supset W\supset T-f(p)$. Therefore $f(\overline{O})\supset T$ and $f(p)$ must be in the interior of $f(\overline{O})$ and thus in the interior of $f(D)$.