BOOK REVIEWS

Riemann surfaces. By Lars V. Ahlfors and Leo Sario. Princeton University Press, Princeton, New Jersey, 1960. 11+382 pp. $10.00.

For the reader who wants to become acquainted with Riemann surfaces, this book offers an excellent survey of the modern theory. Although it is written as a self-contained treatment for the beginner, it is a pleasure for those already familiar with the subject to read this book, for it is written with the elegance and subtlety that also characterize the *Complex analysis* book by Ahlfors. The book moves at a rather lively pace and one is not encumbered by details that can be supplied by the reader.

The authors' main purpose seems to be the presentation of some of the recent developments in the theory of open Riemann surfaces. As preparation for this, the reader must receive a thorough grounding in topology, and this is just what the first and longest of the five chapters provides. The basic concepts of point set topology are reviewed and surfaces and bordered surfaces are defined. The fundamental group, the index of a curve, the degree of a mapping, and orientability of a surface are introduced and a rather extensive treatment of covering surfaces follows. Simplicial homology theory is presented and the invariance of the 1-dimensional homology group is proved by showing that it is isomorphic to the 1-dimensional singular homology group, which in turn is shown to be isomorphic to the abelianized fundamental group. There follows a very interesting discussion of a compactification of an open surface which distinguishes between the different "components" of the ideal boundary. After classifying polyhedra, a proof of the triangulability of every countable surface is given.

It is not until Chapter 2 that a Riemann surface is defined. The authors present two different proofs of the existence of harmonic and analytic functions on a Riemann surface. In Chapter 2, subharmonic functions are used in the solution of the Dirichlet problem for relatively compact regions with non-empty boundary. This is enough to enable the authors to prove that every Riemann surface is countable, i.e., has a countable base. The general existence theorems for harmonic functions with prescribed behavior is given in Chapter 3 using the method of normal operators of Sario. The canonical mappings of planar (schlichtartig) Riemann surfaces are included along with a discussion of the capacity of boundary components of a Riemann surface.