ON THE DETERMINANTS OF CERTAIN TOEPLITZ MATRICES

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With a function of the form

$$\phi(\theta) + g(x) = \sum_{\nu=-\infty}^{\infty} a_\nu e^{i\nu\theta} + g(x), \qquad 0 \leqq x \leqq 1,$$

we associate for each $n = 0, 1, 2, \cdots$, a Toeplitz matrix

$$T_n(\phi(\theta) + g(x)) = \left\{a_{i-j} + \delta_{ij} g\left(\frac{i}{n+1}\right)\right\}, \qquad i, j = 0, 1, \cdots, n,$$

where $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$.

Furthermore we define

$$D_n(\phi(\theta) + g(x)) = \det T_n(\phi(\theta) + g(x)),$$

$$G(\phi(\theta) + g(x)) = \exp\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}\int_0^1 \log(\phi(\theta) + g(x))dx d\theta\right\},$$

$$L(\phi(\theta) + g(x)) = \lim_{n\to\infty} \frac{D_n(\phi(\theta) + g(x))}{[G(\phi(\theta) + g(x))]^{n+1}},$$

whenever these definitions make sense.

We shall prove that under the conditions

(i) $g(x)$ is real and differentiable for $0 \leqq x \leqq 1$ with $g'(x)$ satisfying the Lipschitz condition

$$|g'(x_1) - g'(x_2)| < K|x_1 - x_2|^\alpha, \qquad K > 0, \alpha > 0,$$

(ii) $\phi(\theta)$ is a trigonometric polynomial of the type

$$\phi(\theta) = \sum_{\nu=-k}^{k} a_\nu e^{i\nu\theta},$$

$$a_0 = 0,\ a_\nu = a_{-\nu},\ a_\nu \text{ real}, \qquad \nu = 1, 2, \cdots, k,$$

(iii)
$$\sum_{\nu=-k}^{k} |a_\nu| < g(x), \qquad \text{for } 0 \leqq x \leqq 1,$$

the limit $L(\phi(\theta) + g(x))$ exists and has the value

(1)
$$L(\phi(\theta) + g(x)) = \left(\frac{G(\phi(\theta) + g(0))}{G(\phi(\theta) + g(1))} L(\phi(\theta) + g(0)) L(\phi(\theta) + g(1))\right)^{1/2}.$$