# A THEOREM ON ACYCLICITY

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THEOREM. *Let $X$ and $Y$ be compact Hausdorff spaces and let $M$ be a set-to-set function which assigns to each closed set $A$ in $X$ a closed set $M(A)$ in $Y$ and with the properties*

(a) *If $A_1$ and $A_2$ are closed subsets of $X$ then $M(A_1 \cup A_2) = M(A_1) \cup M(A_2)$,*

(b) *If $x \in X$ and if $V$ is an open set about $M(x)$ then there is an open set $U$ about $x$ with $M(U^*) \subset V$, the * denoting closure,*

(c) *If $A_1$ and $A_2$ are closed sets in $X$ then there is a closed set $A_3$ in $X$ such that $M(A_1) \cap M(A_2) = M(A_3)$,*

(d) *For any two closed sets $A_1$ and $A_2$ of $X$ the set $M(A_1) \cap M(A_2)$ is connected.*

*If also $Y_0$ is a closed subset of $Y$ and if $H^p(M(x), M(x) \cap Y_0) = 0$ for each integer $p \geqq 1$ and each $x \in X$ then $H^p(M(A), M(A) \cap Y_0) = 0$ for each closed subset $A$ of $X$ and each such integer $p$.*

Although the theorem may, at first glimpse, seem closely related to results of Begle [2] and Vietoris [7], its relationship to propositions of Golab [3], Roberts [4], Rutt [5], and G. S. Young [8] is more immediate. It can be reformulated in several ways and the following corollary to one way of stating it will indicate its connection with a result of Golab's:

COROLLARY. *Let $M$ be an upper semicontinuous point-to-set function which assigns to each point $x$ of a simple closed curve $X$ a continuum containing $x$ which does not cut the plane and suppose, in addition, that all of the sets $M(x)$ have a point in common and that the intersection of any two of them is connected; then their union contains the cell bounded by $X$.*

It should be observed that the hypotheses on $Y$ can be weakened and it is indeed enough to suppose that $Y$ is fully normal, or even less [6; 9].

The condition (c), because of its unusual character, deserves a comment, which can best be put in this way. Suppose that $X$ and $Y$ are compact Hausdorff, that $R$ is a closed subset of $X \times X$ and that $M(A)$ is written for the second projection of $(A \times X) \cap R$; then (c) is satisfied if $R$ is both reflexive and transitive. When the remainder of the hypotheses are fulfilled the theorem can be applied to structs [10] as the following example of its use will show: