A FAMILY OF SIMPLE GROUPS ASSOCIATED WITH THE SIMPLE LIE ALGEBRA OF TYPE (F_4)

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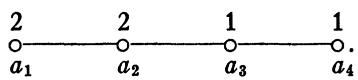
Communicated by Nathan Jacobson, September 12, 1960

In this note we obtain a family of simple groups, which also seem to be new, by applying the method we used in [3] to the Chevalley groups of type (F_4). The orders of the finite groups in the family are

$$q^{12}(q-1)^2(q+1)(q^2+1)(q^3+1)(q^6+1),$$

where $q=2^{2n+1}$, $n=1, 2, 3, \cdots$.

Let $\mathfrak{g}$ be the simple Lie algebra of type (F_4) over the complex number field, and Σ the root system of $\mathfrak{g}$. Let the Coxeter-Dynkin diagram of Σ be



Let P be the additive group generated by Σ, and $\phi: P\to P$ a homomorphism defined by (see [2, Exposé 24, p. 4]),

$$\phi(a_1) = 2a_4, \qquad \phi(a_2) = 2a_3, \qquad \phi(a_3) = a_2, \qquad \phi(a_4) = a_1.$$

Then for any $r\in\Sigma$ we have $\phi(r)=\lambda(r)\bar{r}$, where $\lambda(r)$ is the length of the root r and where $r\to\bar{r}$ is a permutation of order 2 of Σ.

Let K be a field of characteristic 2 which admits an automorphism $t\to t^\theta$ such that $2\theta^2=1$. Define the algebra $\mathfrak{g}_K$ over K and the automorphisms $x_r(t)$, where $r\in\Sigma$, $t\in K$, of $\mathfrak{g}_K$ as in [1], and let G be the group generated by all the $x_r(t)$. Then we have:

(1) The group G admits an automorphism $x\to x^\sigma$ such that

$$x_r(t)^\sigma = x_{\bar{r}}(t^{\lambda(\bar{r})\theta})$$

for all $r\in\Sigma$, $t\in K$.

(2) The group G^1 of all elements x in G such that $x=x^\sigma$ is simple if K has more than two elements.

In order to describe the group G^1 more closely, let $\mathfrak{U}$ be the subgroup of G generated by all the $x_r(t)$ with $r>0$, and set $\mathfrak{U}^1=\mathfrak{U}\cap G^1$. For $r\in\Sigma$, $r>0$, $\lambda(r)=1$, set

$$\alpha(t) = \begin{cases} x_r(t^\theta)x_{\bar{r}}(t) & \text{if } r+\bar{r}\notin\Sigma. \\ x_r(t^\theta)x_{\bar{r}}(t)x_{r+\bar{r}}(t^{\theta+1}) & \text{if } r+\bar{r}\in\Sigma. \end{cases}$$

[1] This work was done while the author held a Research Associateship of the Office of Naval Research, U. S. Navy.