DIFFERENTIABLE IMBEDDINGS

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1. **Terminology.** V^n and M^m will be differentiable manifolds of dimension n and m respectively; differentiable meaning always of class C^∞. For simplicity, we assume V compact and without boundary.

We shall have to consider several categories of maps:

(1) the category of continuous maps,

(2) the category of topological imbeddings,

(3) the category of topological immersions: a map $f: V \to M$ is a topological immersion of V in M if the restriction of f to some neighborhood of each point of V is an imbedding,

(4) the category of differentiable immersions: a map $f: V \to M$ belongs to this category if f is differentiable of rank $n = \dim V$ everywhere,

(5) the category of differentiable imbeddings: a differentiable imbedding $f: V \to M$ is a topological imbedding which is also a differentiable immersion.

Two maps $f_0, f_1: V \to M$ in one of the preceding categories are said to be *homotopic* in this category, if there exists a map $F: V \times R \to M$ (called a *homotopy* from f_0 to f_1) such that $F | V \times \{0\} = f_0$, $F | V \times \{1\} = f_1$ and the associated map $(x, t) \to (F(x, t), t)$ of $V \times R$ in $M \times R$ belongs to the given category.

A homotopy in the category of differentiable imbeddings is also called a *differentiable isotopy* (cf. [4]).

2. **Existence theorem.** Many results have been obtained recently in the combinatorial case (cf. [2; 3; 10; 12; 13]).

The following theorem is in some sense a generalization of Whitney's theorems (cf. [6; 8]) and Wu's theorem (cf. [11]) and the differentiable analogues of the above results.

A space X is q-connected (q an integer) if its homotopy groups vanish in dimension less or equal to q (for $q < 0$, the condition is empty; for $q = 0$, X is connected; for $q = 1$, X is connected and simply connected, and so on).

THEOREM 1. *Let V^n and M^m be two differentiable manifolds which are respectively $(k-1)$-connected and k-connected. Then*

(a) *Any continuous map of V in M is homotopic to a differentiable*

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