

The theory of matrices. By F. R. Gantmacher. Trans. from the Russian by K. A. Hirsch, vols. I and II. New York, Chelsea, 1959. 10+374 pp.; 9+276 pp. \$6.00 each.

Applications of the theory of matrices. By F. R. Gantmacher. Trans. from the Russian by J. L. Brenner, with the assistance of D. W. Bushaw and S. Evanusa. New York, Interscience, 1959. 9+317 pp. \$9.00.

The original edition of Gantmacher's *Teoriya Matrits* consists of a single volume with fifteen chapters. The Chelsea volumes are translations of Chapters I through X and Chapters XI through XV respectively. The Interscience volume is a translation of Chapters XI through XV.

In the first ten chapters the reader is introduced to the classical matrix theory in the classical way. The following brief summary may give an indication of the author's point of view and the scope of the treatment: The book begins with a definition of matrices and the usual manipulations associated with them; the Gauss elimination scheme is studied in great detail. Next, vector spaces and linear transformations are introduced and the connection with matrices is given. This is followed by a thorough treatment of the characteristic and minimal polynomial and functions of matrices. Two independent treatments of similarity theory are given—one based on matrices with polynomial entries and the other (as Gantmacher puts it, after Krull) based on the decomposition of a vector space into cyclic subspaces relative to a linear transformation. Now one is ready to study matrix equations and among those treated are $AX - XB = C$, $X^m = A$, and $e^X = A$. The last two chapters are devoted to that part of matrix theory that has grown out of the study of quadratic and hermitian forms. Unitary and euclidean spaces are defined and hermitian, unitary, symmetric, skew-symmetric, orthogonal, and normal linear transformations are studied; the canonical forms for the corresponding matrices are obtained and some of the standard factorization theorems appear. The reduction of quadratic forms to sums of squares is given detailed treatment; and the extremal properties of the eigenvalues of a regular pencil of forms are studied; Chapter X closes with a section on Hankel forms.

There are no exercises for the reader although there are many worked-out numerical examples—particularly in the treatment of similarity theory. There are several applications to differential equations. No mention is made of dual spaces or tensor products—a knowledge of elementary determinant theory is a prerequisite.

The author has given a masterful presentation of this particular