RECENT PROGRESS IN ERGODIC THEORY[1]

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Prologue. In 1948, at the November meeting of the Society in Chicago, I delivered an address entitled *Measurable transformations.* In the twelve years that have elapsed since then, ergodic theory (of which the theory of measurable transformations is the greatest part) has been spectacularly active. The purpose of today's address is to report some of the developments of those twelve years; its title might well have been *Measurable transformations revisited.* The subjects I chose for this purpose are: some new ergodic theorems, information theory and its connection with ergodic theory, and the problem of invariant measure.

The stage on which most ergodic performances take place is a measure space consisting of a set X and of a measure μ defined on a specified σ-field of measurable subsets of X. At the most trivial level X consists of a finite number of points, every subset of X is measurable, and μ is a mass distribution on X (which may or may not be uniform). At a more useful and typical level X is the real line $(-\infty, +\infty)$, or the unit interval $[0, 1]$, measurability in either case is interpreted in the sense of Borel, and μ is Lebesgue measure. Another possibility is to consider a measure space having a finite number of points with total measure 1 and to let X be the Cartesian product of a countably infinite number of copies of that space with itself; measurability and measure in this case are interpreted in the customary sense appropriate to product spaces. This latter example is easily seen to be measure-theoretically isomorphic to the unit interval, as also are most of the normalized measure spaces (measure spaces with total measure 1) that ever occur in honest analysis. The only measure spaces I shall consider in this report are the ones isomorphic to one of the spaces already mentioned. The expert will know just how little generality is lost thereby, and the casual passer-by, quite properly, will not care.

A transformation T from a measure space X into a measure space Y is called *measurable* if the inverse image $T^{-1}E$ (in X) of each measurable set E (in Y) is again a measurable set. A measurable transformation T is *measure-preserving* if, for every measurable set E, the sets E and $T^{-1}E$ have the same measure. A measurable (but not

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