

A FAMILY OF SIMPLE GROUPS ASSOCIATED WITH THE SIMPLE LIE ALGEBRA OF TYPE $(G_2)^1$

BY RIMHAK REE

Communicated by Nathan Jacobson, August 15, 1960

The purpose of this note is to announce the construction of a family of simple groups which seem to be new. The family contains finite as well as infinite groups. The orders of the finite groups in the family are $q^3(q-1)(q^3+1)$, where $q=3^{2n+1}$, $n=1, 2, 3, \dots$. The construction is carried out by applying to the Chevalley groups of type (G_2) a method which emerges naturally when one looks at Suzuki's [3] construction of his simple groups from the Lie-theoretical point of view.

For the sake of the reader who is not familiar with the Lie theory, we shall define, in the last paragraph of this paper, the groups purely in terms of matrices.

Let \mathfrak{g} be the simple Lie algebra of type (G_2) over the complex number field, and $\Sigma = \{\pm a, \pm b, \pm(a+b), \pm(2a+b), \pm(3a+b), \pm(3a+2b)\}$ the set of roots of \mathfrak{g} . Let \mathfrak{g}_K be the corresponding algebra over an arbitrary field K , and define automorphisms $x_r(t)$, where $r \in \Sigma, t \in K$, of \mathfrak{g}_K as in [1]. Denote by G the group generated by the $x_r(t), r \in \Sigma, t \in K$. Let P be the additive group generated by Σ , and K^* the multiplicative group of nonzero elements in K . As in [1], associate with each homomorphism $\chi: P \rightarrow K^*$ an automorphism $h(\chi)$ of \mathfrak{g}_K . It is known [1] that G contains all the $h(\chi)$. Also, G contains an element ω_0 such that $\omega_0^2 = 1, \omega_0 x_r(t) \omega_0^{-1} = x_{-r}(-t)$ for all $r \in \Sigma, t \in K$.

Now let K be a field of characteristic 3 which has an automorphism $t \rightarrow t^\theta$ such that $3\theta^2 = 1$, the identity automorphism. Note that any finite field of $q = 3^{2n+1}$, ($n \geq 1$), elements has the above property with $t^\theta = t^m$, where $m = 3^n$. Now define the group G^1 to be the subgroup of G generated by ω_0 and the elements of the form

$$\alpha(t) = x_a(t^\theta)x_b(t)x_{a+b}(t^{\theta+1})x_{2a+b}(t^{2\theta+1}), \quad t \in K.$$

Then we have:

- (1) *If K has more than three elements, then G^1 is simple.*
- (2) *The subgroup U^1 generated by the $\alpha(t), t \in K$, contains all elements of the forms*

¹ This work was done while the author held a Research Associateship of the Office of Naval Research, U. S. Navy.