RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

POLYHEDRAL HOMOTOPY-SPHERES

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Communicated by Edwin Moise, July 18, 1960

It has been conjectured that a manifold which is a homotopy sphere is topologically a sphere. This conjecture has implications, for example, in the theory of differentiable structures on spheres (see, e.g., [3, p. 33]).

Here I shall sketch a proof of the following theorem:

Let M be a piecewise-linear manifold of dimension $n \geqq 7$, which has the same homotopy-type as the n-sphere S^n. Then there is a piecewise-linear equivalence of $M - \{point\}$ with euclidean n-space; in particular, M is topologically equivalent to S^n.

This theorem is not the best possible, for C. Zeeman has been able to refine the method presented here so as to prove the same theorem for $n \geqq 5$.

A *piecewise-linear n-manifold* is a polyhedron with a linear triangulation satisfying the condition that the link of each vertex is combinatorially equivalent to the standard $(n-1)$-sphere; all the manifolds with which I am concerned here have no boundary. In general, all the spaces in this paper will be polyhedra, finite or infinite, and each map will be polyhedral, i.e., induced by a simplicial map of linear triangulations.

Let K be a finite subpolyhedron of the finite polyhedron L; let K' be a finite subpolyhedron of the finite polyhedron L'; let $f: L \rightarrow L'$ be a polyhedral map. f is called a *relative equivalence* $(L, K) \Rightarrow (L', K')$, if $f(K) \subset K'$ and $L - K$ is mapped by f in a 1-1 manner onto $L' - K'$.

Recall J. H. C. Whitehead's definition of contraction [7, p. 247]: If the simplicial complex A has a simplex σ^p which is the face of just one simplex τ^{p+1}, and B is the simplicial complex obtained from A by removing the open simplexes σ^p and τ^{p+1}, then $A \rightarrow B$ is called an *elementary contraction* at σ^p. A finite sequence of elementary contractions is a *contraction*.

If K is a finite subpolyhedron of the finite polyhedron L, then it is said that L *contracts onto* K, if there is a linear triangulation A of L,

[1] The author has a fellowship of the National Science Foundation, U.S.A.