BOOK REVIEWS

Nonlinear problems in random theory. By Norbert Wiener. The Technology Press of Massachusetts Institute of Technology, 1958. 9+131 pp. $4.50.

Professor Wiener gave a seminar of fifteen lectures to a group of faculty and graduate students in Electrical Engineering at the Massachusetts Institute of Technology in 1958. This little book consists of that set of lectures, according to the author's preface, set down almost verbatim. The central idea of these lectures is the systematic use of polynomial functionals of Brownian motion in the solution of nonlinear problems from electrical engineering, physics, and elsewhere. An unusual variety of applications is considered, including frequency modulation, brain waves, coupled oscillators, nonlinear electrical networks, coding and decoding (in the information-theoretic sense), quantum theory, and statistical mechanics. The style of the book is highly informal. Informality of itself is perhaps refreshing, but here the reviewer feels that too often the informality has degenerated to sloppiness. One of several noticeable examples is a phrase appearing on page 45, "$\cdots$ since $F(T^t\alpha)\overline{F(\alpha)}$ is obviously a function which will be L^2, being the product of two functions in L^2." A more careful mathematical proofreading would have been desirable.

The first four lectures, comprising about the first third of the book, are used to develop heuristically the necessary theory for the applications to follow. A brief, easily readable, intuitive account of Wiener measure and the Wiener integral, patterned somewhat after a section of the author's Acta Mathematica paper of 1930, is given in the first lecture. Following this, polynomial functionals of Brownian motion are introduced (as e.g. $\int \cdots \int K(\tau_1, \cdots, \tau_n)dx(\tau_1, \alpha) \cdots dx(\tau_n, \alpha)$ where $dx(t, \alpha)$ denotes integration with respect to τ, and α is the stochastic variable). The calculus for finding the averages of such polynomials is demonstrated, and a reduction to a canonical form is developed, whereby to each symmetric kernel $K(\tau_1, \cdots, \tau_n)$ in L_2 is assigned an nth degree nonhomogeneous polynomial functional orthogonal to all polynomial functionals of lower degree. This reduction permits expansion of general nonlinear functionals in L_2 (Wiener measure) in orthogonal series of polynomial functionals (closely related to expansions obtained by Cameron and Martin in 1947), and is the essential tool used throughout the book.

We shall mention here just a couple of the applications. In the lectures on frequency modulation and brain waves, there is a discussion of the functionals