BOOK REVIEWS

Ramification theoretic methods in algebraic geometry. By Shreeram Abhyankar, Princeton, Princeton University Press, 1959. 7+96 pp. $2.75.

Abhyankar uses certain special definitions. A *local ring* (R, M) is any ring R with unit having a single maximal ideal M. Here R need not be Noetherian. A *semi-local ring* $(S: M_1, \cdots, M_t)$ is defined similarly. Finally, a domain A is called *normal* if it is integrally closed in its quotient field.

The basic situation of the book finds a normal local domain (R, M) with field of quotients K, and a finite algebraic extension K' of K. Then the integral closure S of R in K' is a semi-local domain $(S: N_1, \cdots, N_t)$ with a finite number of maximal ideals $N_1, \cdots, N_t$. The ideal MS is given by an expression of the form $MS = Q_1 \cap \cdots \cap Q_t = Q_1 \cdot \cdots \cdot Q_t$, where $Q_1, \cdots, Q_t$ are primary for $N_1, \cdots, N_t$ respectively. The normal local domains $R_1, \cdots, R_t = S_{N_1}, \cdots, S_{N_t}$ are said to *lie over* R. For each $i = 1, \cdots, t$, $R_i \cap K = R$. So R, which is uniquely determined by R_i, is said to *lie below* R_i. Let $M_i = N_i R_i$ be the maximal ideal of R_i. Then (R_i, M_i) is said to be *unramified* over (R, M) if the following two conditions are satisfied:

$$(1) \qquad \begin{aligned} &\text{(a) } R_i/M_i \text{ is a separable extension of } R/M, \\ &\text{(b) } MR_i = M_i. \end{aligned}$$

Otherwise (R_i, M_i) is *ramified* over (R, M). The integral closure S is *unramified* over R if *all* the domains lying over R are unramified; otherwise it is ramified.

If S' is any domain with quotient field K' such that $R \subset S' \subset S$, then the *discriminant ideal* $D(S'/R)$ is defined to be the ideal of R generated by all the discriminants $D_{K'/K}(w_1, \cdots, w_n)$, where $w_1, \cdots, w_n$ is any basis of K'/K lying in S'. The domain S' is also semi-local with, say, maximal ideals $M_1', \cdots, M_s'$. We have then the general discriminant theorem of Krull which states that:

$$\sum_{i=1}^{S} [(S'/M_i'):(R/M)]_S \leqq [K':K]$$

with equality if and only if $D(S'/R) = R$. In the latter case, $S' = S$, S is a free R-module, and is unramified over R.

Suppose further that K'/K is a Galois extension (i.e., finite normal separable algebraic). Let $G = G(K'/K)$ be its Galois group. Then G permutes the domains $R_1, \cdots, R_t$ transitively. The splitting group