BOOK REVIEWS

The theory of groups. By Marshall Hall, Jr., New York, MacMillan Co., 1959. 13+434 pp. $8.75.

When, in 1911, W. Burnside published the second edition of his *Theory of groups of finite order*, his work contained all the essential group theoretical knowledge of his time with the exception of the theory of continuous groups (as they were called fifty years ago). In spite of the large number of results found and of methods developed since Burnside's time, Hall's *Theory of groups* can claim to be a "Burnside brought up-to-date." Clearly, this cannot mean any more an account of all things known which now would require a book of thousands of pages. But Hall introduces the reader to the most important concepts and methods available in group theory (outside of the theory of Lie groups and topological groups), and he leads him in many cases to the frontier of our knowledge. Almost throughout, complete proofs are given for all results stated as theorems. Notable exceptions are Theorems 17.2.1 (p. 314) on the standard form of an element in a free product with amalgamated subgroups and 18.4.8 (p. 337) on the complete solution of Burnside's problem for exponent 6 by Higman, P. Hall and the author. However, it is not astonishing that these exceptions occur but that they are so rare.

The first four chapters of the book deal with definitions, homomorphisms, direct and Cartesian products, Sylow theorems, elementary results on abelian groups and the basis theorem for finite abelian groups. Although all of this is standard material which has been presented many times in many places, the author approaches it in a new and very efficient manner; at the end, he is able to discuss the groups of order p, p^2, pq, p^3 in three pages. Abelian groups are taken up again and discussed more fully in Chapter 13.

The fifth chapter on permutation groups is a brief but important account of this subject, including a discussion of the alternating group and a proof of a generalization of a theorem of Jordan referring to four-fold transitive groups. The chapter ends with a discussion of the wreath product and the Sylow subgroups of the symmetric groups.

Chapter 6 on automorphisms and Chapter 8 on lattices and composition series contain basic facts; the topic of Chapter 8 is taken up again in Chapter 14 on lattices and subgroups. It ends with the proof of a theorem due to Iwasawa which characterizes as supersolvable