SPACES OF RIEMANN SURFACES AS BOUNDED DOMAINS[1]

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1. A *surface of type* (g, n) is a surface S obtained by removing $n \geqq 0$ distinct points from a closed Riemann surface of genus g; we assume always that $3g-3+n>0$. Such a surface S is *marked* by choosing a homotopy class $[f]$ of orientation preserving homeomorphisms f of S onto a fixed *reference surface* S_0. Two marked surfaces $(S_1, [f_1])$ and $(S_2, [f_2])$ are *equivalent* if there is a conformal mapping g with $g(S_1)=S_2$, $[f_2 g]=[f_1]$. *The Teichmüller space* $T_{g,n}$ is the set of equivalence classes of marked surfaces of type (g, n). It is known that $T_{g,n}$ carries two natural structures: that of a *metric space*, homeomorphic to a $(6g-6+2n)$-cell [**1**; **4**; **10**; **11**], and that of a *complex manifold* [**2**; **5**; **7**; **8**; **12**].

We sketch in §§2–7 a proof of the

THEOREM. *$T_{g,n}$ is (holomorphically equivalent to) a bounded domain in complex number space.*

In §§8–10 we consider factor spaces of $T_{g,n}$.

2. A group G of Möbius transformations is called a *Schottky group* of genus g if there exists a closed region R (*standard fundamental region*) on the Riemann sphere P, bounded by $2g$ disjoint Jordan curves C_j, and g elements $A_1, \cdots, A_g$ of G (*standard set of generators*) with $A_j(C_{2j-1})=C_{2j}$. In this case the A_j generate G, the limit points of G form a perfect nowhere dense set $Q(G)$ of measure zero, G is properly discontinuous and fixed-point-free on $P-Q(G)$, and $(P-Q(g))/G$ is a closed Riemann surface of genus g.

Every closed Riemann surface can be so represented.

This classical theorem (Klein-Koebe-Courant) can be proved as follows. Let G_0 be a *fixed* Schottky group and S a *given* closed Riemann surface of the same genus. Then S can be represented as the surface $(P-Q(G_0))/G_0$ on which the conformal structure has been redefined by means of a Riemannian metric $ds=|dz+\mu(z)d\bar{z}|$, where $\mu(z)$ is measurable, $\mu(z)dz/d\bar{z}$ is invariant under G_0 and $|\mu(z)| \leqq k<1$ (cf. [**3**; **4**]). Let $z \rightarrow W(z)$ be the uniquely determined homeomorphism

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