A MODIFICATION OF HARSANYI'S BARGAINING MODEL

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1. **Introduction.** J. C. Harsanyi has described [**1**] a bargaining model for an arbitrary game, which he treats by an extension of the bargaining theory of Nash [**3**] to obtain a unique imputation called the *solution.* I believe there are very serious objections to Harsanyi's model, of which the following is the most convincing. One can describe a three-person game (see later) in which any one player, if faced with a coalition of the other two, can get 10; but the Harsanyi solution is (15, 15, 9).

Harsanyi's model, treated by the bargaining theory advanced in my thesis [**2**], still yields paradoxical solutions. Perhaps there is an irremediable defect in the model. However, on the other side, Harsanyi has shown that applying the model to a game with linearly transferable utility and constant sum yields the Shapley value. This result is more or less independent of the choice of bargaining theory, and suggests that the model is at least worth considerable further study.

This note describes a bargaining theory which can be applied to the Harsanyi model to yield a unique imputation associated with every game; this imputation gives every player at least as much as he could get from playing the game against a coalition of all other players. The main ingredient in the theory is the arbitration scheme (in the technical sense [**5**]). This is the simple scheme which associates to the threat point $(x_1, \cdots, x_n)$ the highest feasible point of the form $(x_1+a, x_2+a, \cdots, x_n+a)$.

I include a list of axioms which suffice to characterize this arbitration scheme, and descriptions of two test examples. Details will be published elsewhere if the theory survives criticism.

2. **The modified model.** For simplicity I shall treat only games in which there is no problem of strategy; the methods for passing from this case to the general case, since the work of Nash [**4**] and others [**1**; **2**; **5**], are routine, and since the foundations of the theory are to be examined it is desirable not to bury them too deeply. Precisely, consider an *end game* $\ni$ in the sense of [**2**]. A pidgin version of the definition of an end game will suffice. We have a nonempty finite set N of players i; with each player i there is associated a *utility space*

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