RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

A CANONICAL FORM FOR AN ANALYTIC FUNCTION OF SEVERAL VARIABLES AT A CRITICAL POINT[1]

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THEOREM. *Let $f(z, w)$ be analytic in (z, w) for small $|z|$ and $|w|$. Let $n>1$, (since the case $n=1$ is trivial), let*

$$\frac{\partial^k f}{\partial w^k}(0, 0) = 0 \qquad 1 \leqq k < n,$$

and let

$$\frac{\partial^n f}{\partial w^n}(0, 0) \neq 0.$$

Then there is an analytic function g of (z, s) for small $|z|$ and $|s|$ such that setting

$$w = s + s^2 g(z, s) \tag{1}$$

in $f(z, w)$ yields $f(z, w)=P(z, s)$ where P is a polynomial in s

$$f(z, w) = P(z, s) = \sum_{=0}^{n} p_j(z)s^j. \tag{2}$$

The p_j are analytic for small $|z|$,

$$p_j(0) = 0 \qquad 1 \leqq j \leqq n-1,$$

and $p_n(0)\neq 0$. Clearly of course (1) implies $s=w+w^2j(z, w)$ where h is analytic for small $|z|$ and $|w|$. Thus for any small z there is a one to one analytic correspondence between w and s for small $|w|$ and $|s|$.

The result (2) is somewhat reminiscent of the Weierstrass preparation theorem but is different in that the polynomial on the right of (2) is not multiplied by a function of (z, s). On the other hand to achieve the canonical polynomial (2), it is necessary to use the change of variables (1).

A case of this theorem where $n=3$ arises in the transformation of confluent saddle points to Airy integrals [**1**; **2**] and is treated there.

An indication of the proof of the theorem follows. Since one can take $p_0(z)$ in (2) as $f(z, 0)$ there is no loss of generality in treating the

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