CONFORMAL TRANSFORMATIONS OF KAEHLER MANIFOLDS

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1. By studying the *pure* conformal transformations of compact Kaehler manifolds[2] Lichnerowicz [**3**] was able to prove that the largest connected Lie group G of conformal transformations of an Einstein-Kaehler space $M^{2k}(k>1)$ of positive Ricci curvature leaves invariant the Kaehlerian structure.[3] Indeed, the Lie algebra L of G may be expressed as the direct sum of the subalgebra K of the infinitesimal motions and of the space of the pure infinitesimal conformal transformations. Moreover, the pure forms on a compact Kaehler manifold are harmonic and, therefore, since the Ricci curvature is positive, it follows from a result of Bochner that there are no pure conformal transformations [**1**]. The above result holds also in case the Ricci curvature vanishes. Hence, since there are no infinitesimal conformal transformations if the curvature is negative the above statement holds for any compact Einstein-Kaehler space. We remark, however, that a complete Einstein space which admits a global 1-parameter group of conformal transformations must necessarily be a simply connected space of positive constant curvature [**4**]. The result of Lichnerowicz is particularly interesting in the study of homogeneous spaces, for, every homogeneous Kaehler space admits, if the group G is semi-simple, an invariant Einstein-Kaehler metric.

The invariance of the Kaehler structure follows from Lemma 1. However, we prove that the harmonic forms of degree $p=n/2$ of a compact and orientable Riemannian manifold M^n of even dimension n are invariant under the Lie algebra L and it is this fact which allows us to prove that

The largest connected Lie group of conformal transformations of a $4k$*-dimensional compact Kaehler manifold coincides with the largest connected group of automorphisms.*

By an *automorphism* of a Kaehler manifold is meant an analytic homeomorphism of the manifold on itself which is an isometry. The proof depends essentially on formula (6) below.

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[2] The manifolds, differential forms and tensorfields considered are assumed to be of class C^∞.

[3] This was also obtained by Yano and Nagano.