THE EQUIVALENCE OF FIBER SPACES AND BUNDLES[1]

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1. **Introduction.** The objective of this paper is to verify the conjecture made in [2] that every Hurewicz fibration [3] over a polyhedral base is fiber homotopy equivalent to a Steenrod fiber bundle [6]. The result relies heavily on Milnor's universal bundle construction [4] and the following extension [2] of a theorem of A. Dold [1].

THEOREM. *If* $\{E_1, p_1, X\}$ *and* $\{E_2, p_2, X\}$ *are Hurewicz fibrations over a connected CW-complex* X *and if* $f: E_1 \to E_2$ *is a fiber-preserving map such that* f *restricted to some fiber is a homotopy equivalence, then* f *is a fiber homotopy equivalence.*

2. **The associated bundle.** Let $\pi: E \to X$ denote a map, where X is a connected, locally finite polyhedron. Furthermore following Milnor's notation in [4], let $\tilde{S}$, $\tilde{E}$, $\tilde{G}$ denote, respectively, the simplicial paths in X, the simplicial paths emanating from a fixed vertex v_0 and the simplicial loops at v_0. If $\alpha = [x_n, \cdots, x_0]$ is a simplicial path in X we will find it convenient to set $\alpha(0) = x_0$, $\alpha(1) = x_n$. Now, define

$$\Omega_\pi = \{(e, \alpha) \in E \times \tilde{S} \mid \pi(e) = \alpha(0)\}$$

and a map $\xi: \Omega_\pi \to X$ by

$$\xi(e, \alpha) = \alpha(1).$$

Furthermore, let

$$A = \xi^{-1}(v_0) = \{(e, \alpha) \mid \pi(e) = \alpha(0), \alpha(1) = v_0\}.$$

LEMMA. $\{\Omega_\pi, \xi, X, A, \tilde{G}\}$ *is a Steenrod fiber bundle.*

PROOF. Since the proof is entirely analogous to Milnor's proof [4] that $\tilde{E}$ is a bundle over X, we content ourselves with a brief outline. The action $\mu: \tilde{G} \times A \to A$ is defined as follows:

$$\mu[g, (e, \alpha)] = (e, g\alpha).$$

Now, let v_j denote a vertex in X and V_j the star neighborhood of v_j. The coordinate functions

$$\phi_j: V_j \times A \to \xi^{-1}(V_j)$$

are defined by

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