OBSTRUCTIONS TO THE SMOOTHING OF PIECEWISE-DIFFERENTIABLE HOMEOMORPHISMS

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Since the publication in 1956 of John Milnor's fundamental paper [1] in which he constructs differentiable structures on S^7 nondiffeomorphic to the standard one, several further results concerning differentiable structures have been obtained by Milnor, R. Thom, and others. This paper unifies and extends some of these results within the framework of an obstruction theory.

Two differentiable manifolds M, N (connected, not necessarily compact) will be said to be *combinatorially equivalent* if they possess isomorphic C^2 triangulations. If M and N are diffeomorphic, then they are combinatorially equivalent [5]; we seek a partial converse. Let f denote a linear isomorphism between C^2 triangulations of the n-manifolds M and N; we attempt to redefine f in neighborhoods of the open simplices of M, beginning with dimension $n-1$ and working down, so as to make f differentiable. After one step, f is no longer a linear isomorphism between C^2 triangulations; hence one must formulate more general conditions for the induction hypothesis of this step-by-step procedure. The homeomorphism $f: M \to N$ is called a *diffeomorphism mod L*, where L is the m-skeleton of a C^2 triangulation of M, if the following conditions are satisfied:

(1) f is a C^2 diffeomorphism on each closed simplex of L.

(2) f^* is one-to-one on the tangent vectors to L.

(3) The subdivision of M is fine enough that for each simplex σ of L, there are coordinate neighborhoods of $\bar{\sigma}$ and $f(\bar{\sigma})$ in which they are flat.

(4) f is of class C^1 on $M-L$, with Df bounded and $|Df|$ bounded away from zero on any subset of $M-L$ having compact closure. (Df is the Jacobian matrix.)

(5) Let σ be a simplex of L. Choose a coordinate neighborhood of $\bar{\sigma}$ in which it is flat; let z denote coordinates in the plane of $\bar{\sigma}$. Then there is a neighborhood U of σ such that

$$[\partial f/\partial z(p) - \partial f/\partial z(q)]/\|p - q\|$$

and

$$[f(p) - f(q) - Df(p)\cdot(p - q)]/\|p - q\|^2$$

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