FIXED POINTS OF ELEMENTARY COMMUTATIVE GROUPS

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In this Note G is always a compact Lie group. A G-space is a topological space on which G acts continuously. We shall be mainly concerned with the case where G is an *elementary commutative p-group* (p prime or zero), that is, a direct product of a finite number of copies of the circle group T^1 if $p=0$ or of the cyclic group Z_p of order p if p is prime. In this study, a basic role is played by the space X_G, defined in §1. The proofs and additional results are given in the Notes of a seminar on transformation groups, to be published in the Annals of Mathematics Studies.

Although it is not always essential, we assume for simplicity that X is locally compact. $H^i_c(X; L)$ (resp. $H^i(X; L)$) is the ith cohomology group of X with compact (resp. closed) supports, coefficients in L. $\dim_L X$ (resp. $\dim_p X$) is the cohomological dimension [6] with respect to L (resp. a field K_p of characteristic p). The orbit space of X is denoted by X'; $\pi_X: X \to X'$ is the canonical projection, $G_x = \{g \in G,\ g\cdot x = x\}$ the stability group of X and $F(H; X)$ the fixed point set of a subgroup H of G.

1. **The space** X_G. Let X, Y be two G-spaces. The twisted product $X \times_G Y$ is the orbit space of $X \times Y$ under the "diagonal" action $g(x, y) = (g\cdot x, g\cdot y)$. The projections on the two factors induce maps π_1, π_2 of $X \times_G Y$ onto X' and Y' respectively. It is easily seen that given $x' \in X'$ and $x \in \pi_X^{-1}(x')$, the subspace $\pi_1^{-1}(x')$ may be identified with Y/G_x and similarly for π_2. Also, π_2 is a bundle map when Y is a principal G-bundle. We specialize Y to be a universal bundle E_G for G, hence $Y' = E_G/G$ is a classifying space B_G for G.

1.1. LEMMA. *Let $X_G = X \times_G E_G$. Then X_G has projections π_1, π_2 on X' and B_G respectively. π_2 is the projection in a fibre bundle with structural group G and typical fibre X. Let $x' \in X'$, then $\pi_1^{-1}(x')$ may be identified with B_{G_x} ($x \in \pi_X^{-1}(x')$). If G acts trivially on X, then $X_G = X \times B_G$. If $x \in F$, then $\pi_1^{-1}(x) = x \times B_G$ is a cross section for π_2.*

This space has occurred earlier in special cases (see [2; 7; 8] for instance). For discrete G, an algebraic analogue may be found in [11, Chapter V].