

PROPERTIES PRESERVED UNDER ALGEBRAIC CONSTRUCTIONS

R. C. LYNDON

The study of *properties of algebras* belongs equally to Algebra and to Metamathematics. But here we consider only questions that concern all abstract algebras, and we attempt to discuss them without invoking any unfamiliar metamathematical machinery. In consequence, we can not do justice to the many interesting matters that concern important special classes of algebraic systems, nor to the broad metamathematical context in which the problems mentioned here belong.

Our topic was approached in considerable algebraic generality by Philip Hall and his associates; a pioneering result of Garrett Birkhoff, 1935,¹ is mentioned below. A parallel treatment, from a viewpoint presupposing rather novel metamathematical considerations, has been given by P. Jordan, 1949. On the metamathematical side, our topic can be regarded as a chapter in the "theory of elementary (or arithmetical) classes" expounded by Alfred Tarski in 1950, 1954, and, in independent form, by Leon Henkin, 1953, and Abraham Robinson, 1950; most of the recent work discussed below has been directly influenced by these ideas.

AN EXAMPLE. It is a familiar fact that every homomorphic image of a commutative ring is commutative, while a homomorphic image of a domain need not itself be a domain. The proof, that commutativity is preserved under homomorphism and that the property of being a domain is not, is trivial. This triviality resides, at least in part, in the fact that the proof makes no significant reference to the algebraic meanings of the properties in question, but depends rather only on the form of the axioms expressing these properties. If $F(x, y)$ is any polynomial, the same reasoning shows that every homomorphic image of a ring satisfying the condition

$$\text{for all } x \text{ and } y: F(x, y) = 0$$

will have the same property, while the property expressed by the condition

An address delivered before the Evanston Meeting of the Society on November 28, 1958 by invitation of the Committee to Select Hour Speakers for Western Section Meetings; received by the editors December 28, 1958.

¹ A name accompanied by a date will always indicate a reference to the bibliography. The date is that of the publication cited, and not necessarily the year in which the result quoted was first established.