

ON SOME COMPOSITIONS OF HADAMARD TYPE IN CLASSES OF ANALYTIC FUNCTIONS

BY C. LOEWNER AND E. NETANYAHU¹

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1. **Introduction.** In a recent paper by G. Polya and I. J. Schoenberg [1] the following conjecture ascribed to Mandelbrojt-Schiffer was stated:

CONJECTURE I. Let S be the class of regular, and schlicht functions in the unit circle.

Let

$$f(z) = \sum_{n=1}^{\infty} a_n z^n, (a_1 = 1)$$

and

$$g(z) = \sum_{n=1}^{\infty} b_n z^n, (b_1 = 1)$$

be the representing power series of two elements of S . Then the composition

$$(1) \quad h(z) = \sum_{n=1}^{\infty} \frac{a_n b_n}{n} z^n$$

leads to an element of the same class.

A little weaker conjecture than I would be the following:

CONJECTURE II. $h(z)$ has a nonvanishing derivative in $|z| < 1$.

The Conjecture I would mean that under this composition rule S forms a semi-group containing the unit element $z/(1-z)^2$.

The interest in Conjectures I and II lies in the fact that even from the weaker Conjecture II Bieberbach's conjecture would follow immediately.

Indeed by setting

$$g(z) = z + \frac{1}{n} z^n$$

one would obtain the composed function

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