RIEMANN-ROCH THEOREMS FOR DIFFERENTIABLE MANIFOLDS

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1. **Introduction.** The Riemann-Roch Theorem for an algebraic variety Y (see [**7**]) led to certain divisibility conditions for the Chern classes of Y. It was natural to ask whether these conditions held more generally for any compact almost complex manifold. This question, and various generalizations of it, were raised in [**8**] and most of these have since been answered in the affirmative in [**2**] and [**11**].

More recently Grothendieck [**3**] has obtained a more general Riemann-Roch Theorem for a map $f: Y \rightarrow X$ of algebraic varieties. This reduces to the previous Riemann-Roch Theorem on taking X to be a point. Grothendieck's Theorem implies many conditions on characteristic classes, and again it is natural to ask if these conditions hold more generally for almost complex or even differentiable manifolds. The purpose of this note is to enunciate certain differentiable analogues of Grothendieck's Theorem. These "differentiable Riemann-Roch Theorems" yield, as special cases, the divisibility conditions mentioned above and also certain new homotopy invariance properties of Pontrjagin classes. As an application of the latter we get a new proof (and slight improvement) of the result of Kervaire-Milnor [**10**] on the stable J-homomorphism.

Another differentiable Riemann-Roch Theorem, with applications to embeddability problems of differentiable manifolds, will be found in [**1**].

The proofs of our theorems rely heavily on the Bott periodicity of the classical groups [**4**; **5**; **6**], and are altogether different from the earlier methods of [**2**] and [**11**], which were based on Thom's cobordisme theory and Adams' spectral sequence.

2. **Definitions.** The spaces X, Y considered will be countable finite-dimensional CW-complexes, and for simplicity we will suppose them connected.

Following Grothendieck [**3**] we define an abelian group $K(X)$ as follows. Let $F(X)$ be the free abelian group generated by the set of all isomorphism classes of complex vector bundles over X. To every triple $t=(\xi, \xi', \xi'')$ of vector bundles with $\xi \cong \xi' \oplus \xi''$ we assign the element $[t]=[\xi]-[\xi']-[\xi'']$ of $F(X)$, where $[\xi]$ denotes the isomor-