ON A CONJECTURE CONCERNING SCHLICHT FUNCTIONS[1]

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Many years ago and independently of each other S. Mandelbrojt and M. Schiffer were led to the following conjecture, which has appeared in print only recently [2, p. 326]:

CONJECTURE M. S. *If two power series* $\sum_1^\infty a_\nu z^\nu$, $\sum_1^\infty b_\nu z^\nu$ *are schlicht in the unit circle, then also the power series*

$$\sum_1^\infty \frac{a_\nu b_\nu}{\nu} z^\nu$$

is schlicht in the unit circle.

This will be disproved in the following lines. Let D be the image of the unit circle by $w=\sum_1^\infty a_\nu z^\nu$. We denote by the symbols S, Σ and K the classes of such power series for which D is schlicht, schlicht and star-shaped, schlicht and convex, respectively. Evidently $K\subset\Sigma\subset S$.

Observe now that $\sum_1^\infty z^\nu \in K$. By a recent result concerning de la Vallée Poussin means [2, p. 298] we conclude that

$$\sum_1^n \binom{2n}{n+\nu} z^\nu \in K, \qquad (n = 1, 2, \cdots),$$

and therefore [2, Lemma 5, p. 321] that

$$\sum_1^n \nu \binom{2n}{n+\nu} z^\nu \in \Sigma \subset S.$$

Applying the Conjecture M. S. to this special polynomial and an arbitrary power series we obtain the following

COROLLARY OF THE CONJECTURE M.S. *If* $f(z)=\sum_1^\infty a_\nu z^\nu \in S$ *then also*

$$\sum_1^n \binom{2n}{n+\nu} a_\nu z^\nu \in S.$$

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