NORMAL OPERATORS ON THE BANACH SPACE $L^p(-\infty, \infty)$ PART I. BOUNDED OPERATORS[1]

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1. **Introduction.** Let $\mathcal{B}$ denote the Boolean ring generated by the semi-closed intervals of the plane. We denote by $\mathfrak{E}_p$ the algebra of all linear bounded transformations of $L^p(-\infty, \infty)$ into itself. Suppose for a moment that $p=2$, and let $\mathfrak{A}_p$ be an involutive abelian subalgebra of $\mathfrak{E}_p$: if $\mathfrak{A}_p$ is also a Banach space and if $T_p \in \mathfrak{A}_p$, then

(i) *the family of all homomorphic mappings of the ring* $\mathcal{B}$ *into the algebra* $\mathfrak{E}_p$ *contains a member* E_p^T *such that*

$$T_p = \int \lambda \cdot E_p^T(d\lambda). \tag{1}$$

Suppose henceforth that $1 < p < \infty$. To which extent does the preceding situation carry over?

Let $\mathfrak{D}$ be the class of all bounded functions whose real and imaginary parts are piecewise monotone. In §2 will be defined an isomorphism $f \rightarrow \wedge(f)_p$ whose domain includes $\mathfrak{D}$ and whose range $(t)_p$ is a normed[2] involutive abelian subalgebra of $\mathfrak{E}_p$. Our main theorem (§3) shows that a member T_p of $(t)_p$ has the property (i) whenever $T_p = \wedge(f)_p$ for some f in $\mathfrak{D}$. The relation (1) involves a Stieltjes integral defined in the strong operator-topology whenever $p \geqq 2$. The set-function E_p^T need not be countably additive: we do not restrict ourselves to "*spectral resolutions*" in the sense of Dunford (however, condition (ii) in [**1**, p. 219] is satisfied if B and E are replaced by our $\mathcal{B}$ and E_p^T). The values of E_p^T are self-adjoint [**2**, p. 22] idempotent members of $(t)_p$.

It is easily seen that the Hilbert transformation and the Dirichlet operators all have the property (i). For less trivial examples, let $\mathfrak{M}^1$ be the set of all bounded Radon measures on $(-\infty, \infty)$; if $A \in \mathfrak{M}^1$, then the convolution operator A_{*p} is defined as the mapping $x \rightarrow A * x$ of $L^p(-\infty, \infty)$ into itself. In case the Fourier transform of A belongs to $\mathfrak{D}$, then the operator $T_p = A_{*p}$ has the property (i). Consequently,

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[2] In the terminology of Neumark [**5**, p. 110]: the normed ring $(t)_p$ has an isometric involution, and its completion is a (*)-algebra [**2**, p. 22].