

BOOK REVIEWS

Polynomial expansions of analytic functions. By Ralph P. Boas, Jr. and R. Creighton Buck. *Ergebnisse der Mathematik und ihrer Grenzgebiete, New series, No. 19.* Berlin, Springer, 1958, 8 + 77 pp. Paperbound, \$4.75.

This is an excellent little book written with a commendable attention to detail. The authors take pains to indicate frequently what is going on behind the scenes and they include numerous pertinent and useful illustrations of the theory. The book contains much new material in addition to an organized treatment of known phenomena.

The work is motivated by a desire to examine in some detail the fact that certain polynomial sets yield nontrivial representations of zero. Boas and Buck find that, for a specified polynomial set, (a) the nonexistence of such representations of zero, or (b) the existence and enumeration of distinct representations of zero can be deduced from properties of generating functions of certain forms. A very few misprints were noted, none of them likely to disturb the reader.

Chapter I (about 20 pages) contains motivation of the study and some underlying function theoretic results. A set of pseudo Laguerre polynomials (superscript dependent upon the subscript) is used in an illustration. A fairly general class of generating relations previously studied by Boas and Buck is introduced and the corresponding polynomials are given the name generalized Appell polynomials. This class of polynomials is to play a major role in the succeeding chapters. On page 18 two problems, which have long confronted workers in the study of generating relations for polynomial sets, are pointed out to the reader.

Chapter II (about 27 pages) is a study of the representation of entire functions by series of generalized Appell polynomials. There are determinations of the existence of convergent, Mittag-Leffler summable, or Borel summable expansions in such series. Numerous useful illustrations are given. These include polynomials associated with names Bernoulli, Euler, pseudo Laguerre, reversed proper Laguerre (superscript independent of the subscript), Hermite, and a rearranged form of Legendre polynomials. Additional application of much interest is made to the Sheffer polynomials (Sheffer's type zero classification) which include proper Laguerre polynomials, those of Angelescu, Mittag-Leffler and Newton, the actuarial polynomials and some others associated with interpolation problems. Boas and