ALMOST PERIODIC COMPACTIFICATIONS

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1. Introduction. Let B be a Banach space and S a uniformly bounded semigroup of operators on B; i.e., a family S of operators closed under the usual multiplication of operators and satisfying $\|T\| \leqq K$ for some K and all T in S. S will be called *almost periodic* (a.p.) if each orbit is conditionally compact (that is, has compact closure) in the norm topology of B and will be called *weakly almost periodic* (w.a.p.) if each orbit is conditionally compact in the weak topology of B. In either situation, the closure $\bar{S}$ of S in the algebra of bounded operators on B supplied with the weak operator topology is a compact semigroup. Multiplication in $\bar{S}$ is jointly continuous in the a.p. case while only separately continuous in the w.a.p. case.

The purpose of this note is to announce some results which have in common the exploitation of this "compactification" $\bar{S}$ of S.

In §2 we extend the results of Jacobs given in [**8**] and [**9**]. In §3 the theory of weakly almost periodic functions due to Eberlein (see [**3**]) is extended to certain topological semigroups by constructing a compactification that plays the same role in the theory of weakly almost periodic functions as does the Bohr compactification (see [**1**] and [**12**]) in the theory of almost periodic functions on groups. In §4 we study almost periodic functions on semigroups. Our class of functions is wider then that studied by Maak in [**10**] and in the case of the half-line is identical with the class of functions studied by Fréchet in [**6**].

TERMINOLOGY. By a *topological semigroup* we shall mean a semigroup with identity supplied with a Hausdorff topology in which the maps $\tau \rightarrow \sigma \cdot \tau$ and $\tau \rightarrow \tau \cdot \sigma$ are continuous for each σ in S; multiplication is not assumed to be jointly continuous. When we use the term topological group we will mean, as usual, a group supplied with a Hausdorff topology in which multiplication is jointly continuous and inversion is continuous. If S is a topological semigroup, we shall denote by $C(S)$ the Banach space of all complex valued bounded continuous functions on S, supplied with the norm defined by

$$\|f\| = \sup_{\sigma \in S} |f(\sigma)| .$$

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