HOMOMORPHISMS AND IDEMPOTENTS OF GROUP ALGEBRAS

BY PAUL J. COHEN

Communicated by Walter Rudin, January 14, 1959

Let G be a locally compact abelian group. We denote by $M(G)$ the algebra of all finite complex-valued Borel measures on G. The algebra is normed by assigning to each measure its total variation, and the product or convolution of the measures μ and ν is defined by

$$(\mu * \nu)(E) = \int\int_{x+y\in E} d\mu(x)d\nu(y).$$

If a particular Haar measure is chosen on G, the subalgebra of absolutely continuous measures may be identified with $L(G)$, the algebra of absolutely integrable functions. The Fourier transform of a measure μ is a function $\hat{\mu}$ defined on $\hat{G}$, the dual group of G, by the formula

$$\hat{\mu}(\chi) = \int_G (\chi, g)d\mu(g),$$

where (χ, g) denotes χ evaluated at g. Each χ thus yields a homomorphism of $M(G)$ onto the complex numbers. Every such homomorphism of $L(G)$ is obtained in this way.

Let ϕ be a homomorphism of $L(G)$ into $M(H)$. After composing with ϕ, every homomorphism of $M(H)$ onto the complex numbers either is identically zero, or can be identified with a member of $\hat{G}$. We thus have a map ϕ_* from $\hat{H}$ into $\{\hat{G}, 0\}$, the union of $\hat{G}$ and the symbol 0, the latter to be considered as the point at infinity. Our main result is:

THEOREM 1. *For every homomorphism ϕ of $L(G)$ into $M(H)$, there exist a finite number of cosets of open subgroups of $\hat{H}$, which we denote by K_i, and continuous maps ψ_i: $K_i \rightarrow \hat{G}$, such that*

$$\psi_i(x + y - z) = \psi_i(x) + \psi_i(y) - \psi_i(z),$$

with the following property: there is a decomposition of $\hat{H}$ into the disjoint union of sets S_j, each lying in the Boolean ring generated by the sets K_i, such that on each S_j, ϕ_* is either identically zero or agrees with some ψ_i, where $S_j \subset K_i$.

Conversely, for any such map of $\hat{H}$ into $\{\hat{G}, 0\}$, there is a homo-