The hypercircle in mathematical physics. By J. L. Synge. New York, Cambridge University Press, 1957. 12+424 pp. $13.50.

In many fields of mathematical physics, scientists are faced with the problem of solving partial differential equations subject to various boundary conditions, and it is only in rare cases that an exact mathematical solution is feasible. In view of these circumstances, recourse must be had to approximate solutions of one sort or another, e.g. by replacing the differential equation by difference equations to be solved numerically, etc. This book is concerned with the description of a method of approximate solution applicable to a wide class of boundary value problems.

On first glancing upon the intriguing title of the book, the following question arises naturally: Just what is the hypercircle? It is therefore a reviewer's first order of business to attempt to give an answer to this question. This will be done here much in the same way (using Schwarz's and Bessel's inequalities) as it is done in the reviewer's paper in Collectanea Mathematica, Barcelona, mentioned in the preface of the book (this paper was based on an earlier note with A. Weinstein).

Consider a real linear vector space with a scalar product; i.e., a set of elements (called "vectors" following custom) which can be added in pairs ("vector addition"), can be multiplied by real numbers ("scalar multiplication"), these two operations obeying the customary rules of vector algebra. Besides, there is a scalar product, i.e., a real number (a, b) is associated with each ordered pair of vectors a and b, which satisfies the rules: $(\alpha a, b)=\alpha(a, b)$; $(a_1+a_2, b)=(a_1, b)+(a_2, b)$; $(a, b)=(b, a)$, for any vectors a and b and any real number α. If, as will be supposed in most of what follows, $(a, a)\geqq 0$ for any vector a, the scalar product is said to be positive semidefinite. In this case the usual notation $|a|$ is employed for the (non-negative) length of the vector a, and one has $|a|^2=(a, a)$.

The basic inequalities will now be obtained. Let w and c be vectors. Then

$$|w|^2 = |(w-c)+c|^2 = |w-c|^2 + |c|^2 + 2(c, w-c);$$

and in view of Schwarz's inequality: $-|c|\,|w-c| \leqq (c, w-c) \leqq |c|\,|w-c|$, so that

$$\begin{aligned}[|w-c| - |c|]^2 &= |w-c|^2 + |c|^2 - 2|c|\,|w-c| \\ &\leqq |w|^2 \leqq |w-c|^2 + |c|^2 + 2|c|\,|w-c| \qquad (1) \\ &= [|w-c| + |c|]^2.\end{aligned}$$