BOOK REVIEWS

Toeplitz forms and their applications. By Ulf Grenander and Gabor Szegö. California Monographs in Mathematical Sciences, University of California Press, 1958. 7+245 pp. $6.00.

This book owes its timeliness, and much of its importance and unique charm to one particular quality which sets it apart from other research monographs. Its two authors have accomplished a successful synthesis of two important mathematical developments. One of these is the theory of Toeplitz forms, the other, more recent one, the theory of (wide sense) stationary stochastic processes.

The theory of Toeplitz forms has its roots in the work of Toeplitz, Féjèr, Carathéodory, F. Riesz on trigonometric series and harmonic functions. In two important papers (Math. Zeitschrift, 1920) G. Szegö unified and extended much of their work by creating a theory the central ideas and results of which also form the core of the present book. In other words, concepts and methods created forty years ago have now gained new interest as the analytical techniques in a branch of mathematics (prediction and estimation theory for stationary stochastic processes) which did not then exist. This remark could not have been made in 1939, when a brief account of the theory of Toeplitz forms first appeared in Szegö's book *Orthogonal polynomials*, but in any event the present treatment goes further and deeper. We mention in particular the new topics of the trigonometric moment problem (Chapter 4) and a chapter on applications to analytic functions (Chapter 5). To avoid confusion, it should be said at once that the book is divided into two parts; Part I (Chapters 1 to 8) deals with the theory of Toeplitz forms, and Part II (Chapters 9, 10, 11) is devoted to probability and statistics, with the exception of the above mentioned Chapter 9.

In the theory of Toeplitz forms the Fourier Stieltjes coefficients c_k of a distribution function $\alpha(\theta)$ on $[0, 2\pi]$ are used to define the Toeplitz matrices $T_n=(c_{j-i})$, $i, j=0, 1, \cdots, n$. Since $\alpha(\theta)$ is real and nondecreasing, the quadratic forms associated with these matrices are Hermitean positive definite (this restriction, and its regrettable consequences from the point of view of certain applications will form the subject of later remarks). The principal problems of the theory are

(a) *The minimum problem*, which in its simplest form asks for the minimum μ_n of the quadratic form uT_nu where the vector $u=(u_0, u_1, \cdots, u_n)$ is subject to the restriction $u_0=1$. The answer to this famous problem (in Chapter 3) is that the sequence μ_n con-