A GENERALIZED WEYR CHARACTERISTIC[1]

BY A. N. FELDZAMEN

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The classical spectral multiplicity theory for a single normal operator on Hilbert space generalizes the unitary determination of a normal matrix by the multiplicities of its eigenvalues. We outline here the beginnings of an analogous equivalence theory patterned after the similarity determination of an arbitrary (complex) matrix by the Jordan canonical form, or the numerical invariants, the Weyr and Segre characteristics. Generalizations of these characteristics are defined, in spatial rather than combinatorial terms, in the Banach space context, but for application to single operators both abstract structural knowledge of the operators and a multiplicity theory of the classical kind are required; hence our equivalence conclusions are restricted to a class of spectral operators on Hilbert space. Here, the generalized Weyr characteristic provides a complete set of invariants for an equivalence relation slightly more general than similarity. No separability assumptions are required.

We assume various parts of spectral operator (and related) theory outlined in §2 of Dunford's review article [2], including the multiplicity theory of Bade (cf. pp. 235–236 of [2]) for a complete Boolean algebra of projections on a Banach space, and also, the multiplicity theory *in toto* of Halmos [4] for a single normal operator on Hilbert space.

1. **Let** $\mathfrak{X}$ be a Banach space of uniform multiplicity $n<\infty$ with respect to the complete countably-additive spectral measure $E(\cdot)$, defined on the Borel sets $\mathfrak{B}$ of the complex plane $\mathfrak{C}$, with support $\Lambda\in\mathfrak{B}$. (That is, if for $x\in\mathfrak{X}$, $\mathfrak{M}(x)$ denotes clm $[E(\delta)x \mid \delta\in\mathfrak{B}]$, then, whenever $E(\delta)\neq 0$, the space $E(\delta)\mathfrak{X}=\bigvee_{i=1}^{n}\mathfrak{M}(x_i)$ and n vectors are always required.) If f is a Borel function on $\mathfrak{C}$ we write $S(f)$ for the not-necessarily-bounded operator $\int f(\lambda)E(d\lambda)$, and call a set $\delta\in\mathfrak{B}$ an *inverting set* for $S(f)$ if $E(\delta)S(1/f)$ is bounded. Let

$$C(x)=\bigwedge\{E(\delta) \mid E(\delta)x=x,\ \delta\in\mathfrak{B}\}$$

—$C(x)$ is always some $E(\delta)$—and call two sets $\delta,\pi\in\mathfrak{B}$ *equivalent*

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