GENERAL BOUNDARY VALUE PROBLEMS FOR ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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In this paper we combine the methods of Aronszajn and Milgram [3] with those previously employed by the author [9] and solve very general boundary value problems for elliptic equations. For convenience we consider equations, but much of what we say can be carried over to systems without difficulty.

Let $\mu=(\mu_1, \mu_2, \cdots, \mu_n)$ be a sequence of indices, each $\geqq 0$, and set

$$|\mu| = \sum \mu_k, \qquad \xi^\mu = \xi_1^{\mu_1}\xi_2^{\mu_2} \cdots \xi_n^{\mu_n},$$
$$D^\mu = \partial^{|\mu|}/(i\partial x_1)^{\mu_1}(i\partial x_2)^{\mu_2} \cdots (i\partial \chi_n)^{\mu_n}$$

where $\xi=(\xi_1, \xi_2, \cdots, \xi_n)$ is any n-dimensional vector. The linear partial differential operator of order $2r$

$$A = \sum_{|\mu|\leq 2r} a_\mu(x) D^\mu$$

with complex coefficients $a_\mu(x)$ is elliptic in a region $R\subseteq E^n$ if its characteristic polynomial

$$P(x, \xi) = \sum_{|\mu|=2r} a_\mu(x)\xi^\mu$$

does not vanish in R for real $\xi\neq 0$. If R is the closure $\bar{G}$ of a bounded domain G, we shall say that A is properly elliptic in $\bar{G}$ if in addition it satisfies at every point x on the boundary $\dot{G}$ of G (cf. [2; 5; 8]).

CONDITION 1. For every real vector $\tau\neq 0$ parallel to $\dot{G}$ at x and every real $\nu\neq 0$ normal to $\dot{G}$ at x, the polynomial $P(z)=P(x, \tau+z\nu)$ has exactly r roots $\lambda_k(\tau, \nu)$, $k=1, 2, \cdots, r$, with positive imaginary parts.

If $n>2$, all elliptic operators are properly elliptic.

By a boundary operator we shall mean a linear partial differential operator whose coefficients need merely be defined on $\dot{G}$. If

$$B_j = \sum_{|\mu|\leq m_j} b_{j\mu}(x) D^\mu, \qquad j = 1, 2, \cdots, r,$$

is a set of such operators, we set

$$Q_j(x, \xi) = \sum_{|\mu|=m_j} b_{j\mu}(x)\xi^\mu, \qquad j = 1, 2, \cdots, r.$$

We shall say that the set $\{B_j\}_{j=1}^r$ "covers" a properly elliptic operator A if each $m_j<2r$ and the B_j satisfy at each point $x\in\dot{G}$.