

ON OPEN MAPPINGS IN BANACH ALGEBRAS, II

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This paper contains an elementary way to look at the results of [1] and [2]. We prove Theorems 4 and 5 of [2].

Let \mathfrak{A} be a Banach algebra, f a holomorphic function on an open set U in the plane, $a \in \mathfrak{A}$, $\sigma(a) \subset U$, and $f(\sigma(a)) \subset f(V)$ where V is an open set on which f is 1-1. Then a neighborhood of $f(a)$ consists entirely of points of the form $f(b)$ for some $b \in \mathfrak{A}$; further, if $\sigma(a_1) \subset V$ and $f(a) = f(a_1)$, then a and a_1 commute.

To prove this, let \tilde{f} be the restriction of f to V , \tilde{f}^\vee the inverse function to \tilde{f} . \tilde{f}^\vee is defined on an open set W of the plane, with $f(\sigma(a)) \subset W$. There exists a neighborhood of $f(a)$ all of whose elements have spectrum contained in W (see e.g., [2, Lemma 2]). For these elements c , set $b = \tilde{f}^\vee(c)$. Then $f(b) = \tilde{f}(b) = \tilde{f}(\tilde{f}^\vee(c)) = c$.

The second assertion comes from noticing that a commutes with $g(a)$ for any g holomorphic on U ; in particular, with $\tilde{f}^\vee(f(a)) = \tilde{f}^\vee(f(a_1)) = a_1$.

BIBLIOGRAPHY

1. E. Hille, *On roots and logarithms of elements of a complex Banach algebra*, Math. Ann. vol. 136 (1958) pp. 46-57.
2. C. McCarthy, *On open mappings in Banach algebras*, to appear in the Journal of Mathematics and Mechanics.

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