ON THE REFLECTION LAWS OF SECOND ORDER DIFFERENTIAL EQUATIONS IN TWO INDEPENDENT VARIABLES

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The main purpose of this paper is the study of the analytic continuation of solutions of elliptic differential equations in two independent variables across an analytic boundary on which they satisfy further analytic equations connecting the point of the boundary, the value of the solution and those of its first order partial derivatives.

In its simplest form the problem is that of the continuation of a harmonic function of two variables x, y across a segment of the x-axis if on this segment an analytic relation holds between x, the given harmonic function u and its conjugate harmonic v, and the derivatives u_x and u_y. This case was the subject of [2] and of [7]. Here we shall complement the methods and results of [7] by discussing the special case of a linear boundary relation. We shall see that u can be extended as harmonic function into the mirror image of its domain D of definition provided D is simply connected. In other words: the general linear boundary condition and the boundary condition $u=0$ do not differ with respect to the domain into which they permit the class of harmonic functions on D to be extended, they are "coextensive."

In this respect there is a fundamental difference between harmonic functions of two and of three variables. We shall furnish an example of a harmonic function of three variables which satisfies a linear boundary condition, even of constant coefficients, on a boundary plane, but which is not coextensive with the class of harmonic functions of the same domain of definition, but satisfying the condition $u=0$ on the boundary plane. The example is such as to permit the choice of the boundary condition in a manner to exclude an arbitrary point of the mirror image domain from the domain of analytic extension.

This example illustrates, by exhibiting the breakdown of analogy of facts, the necessity of employing in the treatment of equations in two independent variables tools especially adapted to this case. We shall see that in essence the problem of analytic continuation of

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