

Cardinal and ordinal numbers. By W. Sierpiński. Monografie Matematyczne, vol. 34. Warszawa, Państwowe Wydawnictwo Naukowe, 1958. 487 pp.

Not since the publication in 1928 of his *Leçons sur les nombres transfinis* has Sierpiński written a book on transfinite numbers. The present book, embodying the fruits of a lifetime of research and experience in teaching the subject, is therefore most welcome. Although generally similar in outline to the earlier work, it is an entirely new book, and more than twice as long. The exposition is leisurely and thickly interspersed with illuminating discussion and examples. The result is a book which is highly instructive and eminently readable. Whether one takes the chapters in order or dips in at random he is almost sure to find something interesting. Many examples and applications are included in the form of exercises, nearly all accompanied by solutions.

The exposition is from the standpoint of naive set theory. No axioms, other than the axiom of choice, are ever stated explicitly, although Zermelo's system is occasionally referred to. But the role of the axiom of choice is a central theme throughout the book. For a student who wishes to learn just when and how this axiom is needed this is the best book yet written. There is an excellent chapter devoted to theorems equivalent to the axiom of choice. These include not only well-ordering, trichotomy, and Zorn's principle, but also several less familiar propositions: Lindenbaum's theorem that of any two nonempty sets one is equivalent to a partition of the other; Vaught's theorem that every family of nonempty sets contains a maximal disjoint family; Tarski's theorem that every cardinal has a successor, and other propositions of cardinal arithmetic; Kurepa's theorem that the proposition that every partially ordered set contains a maximal family of incomparable elements is an equivalent when joined with the ordering principle, i.e., the proposition that every set can be ordered. It is shown that the ordering principle can be deduced from the existence for any set of a function associating with each subset having at least two elements one of its non empty proper subsets. Included also is the author's deduction of the axiom of choice from the generalized continuum hypothesis, and Mostowski's theorems concerning the restriction of the axiom of choice to families of sets of n elements. However, topological equivalents such as Tychonoff's theorem are not discussed.

The first five chapters deal with sets and elementary operations, equivalence and comparison of sets, denumerable sets, and sets of power c . The axiom of choice is introduced in Chapter 6, but the dis-