SECONDARY OBSTRUCTIONS FOR FIBRE SPACES

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1. The aim of this note is to set up machinery for the computation of the second obstruction cohomology class to the extension of a cross-section of a fibre space (in the sense of Serre [6]), $F \to^{i} E \to^{p} B$, with base B a simply connected C-W complex, and connected fiber F, total space E, projection map p and inclusion i. (We will use this notation below to identify the component parts of fibre spaces.) The problem is reduced to the computation of the Eilenberg-McLane invariant of F, just as for the obstruction problem for mapping $B \to F$, [2], and additional twisting invariants of the fibre space. It is not yet clear how easily these latter invariants can be computed; in simple cases (for example, if F is a sphere [4]), the method works very elegantly. In this example the simplifying points seem to be that (1) the second nonvanishing homotopy group of the fibre is cyclic of prime order and (2) the additive cohomology of E can be explicitly determined.

2. The calculation is based on a construction introduced by J. C. Moore [5] in the category of semi-simplicial fibre spaces giving a decomposition of E by successively killing off homotopy groups of the fibre.

Suppose $0 < n(1) < n(2) < \cdots$ are the dimensions in which F has non-zero homotopy groups. Define $A_j = \pi_{n(j)}(F)$. Recall that the Postnikov system of F provides a tower of fibre spaces (and commutative diagrams)

$$\begin{tikzcd}
 & K(A_i, n(i)) \arrow[d] & & & & \\
\cdots \arrow[r] & F_i \arrow[rr] & & F_{i-1} \arrow[r] & \cdots \arrow[r] & F_1 \\
 & & F \arrow[ul, "g_i"] \arrow[ur, "g_{i-1}"'] & & &
\end{tikzcd} \tag{2.1}$$

such that the map g_i induces an isomorphism on homotopy groups in dimensions $\leqq n(i)$. (If A is an abelian group, $K(A, n)$ is a space all of whose homotopy groups are zero but the one in dimension n, which is isomorphic to A.)

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