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FUNCTIONS WHOSE PARTIAL DERIVATIVES ARE MEASURES

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Let x denote a generic point of euclidean N-space $R^N (N \geqq 2)$. We consider the space $\mathfrak{F}$ of all summable functions $f(x)$ such that the gradient grad f (in the distribution theory sense) is a totally finite measure. $I(f)$ denotes the total variation of the vector measure grad f. In case grad f is a function F we have

$$I(f) = \int_{R^N} | F(x) | \, dx.$$

We write H_k for Hausdorff k-measure; and fr E for the frontier of a set E. Fr E is *rectifiable* if it is the Lipschitzian image of a compact subset of R^{N-1}.

One ought to be able to determine the primitive f with greater precision than grad f, at least in certain cases. Our main result is that indeed f can be determined up to H_{N-1}-measure 0 in two (quite opposed) cases: (1) grad f is a function; (2) the range of f is a discrete set, which we may take to be the integers. More precisely, let $\mathfrak{F}_1$, $\mathfrak{F}_2$ be the sets of those $f \in \mathfrak{F}$ satisfying (1) and (2) respectively. Let $\mathfrak{F}_{01}$ be the set of all Lipschitzian functions f with compact support. Let $\mathfrak{F}_{02}$ be the set of all functions f with the following property: there exist a closed oriented $(N-1)$-polyhedron A and a Lipschitzian mapping $g(w)$ from A into R^N such that, for every $x \notin g(A)$, $f(x)$ is the degree of the mapping g at x, and $f(x) = 0$ for $x \in g(A)$. Write $J(w)$ for the Jacobian vector of $g(w)$, wherever it exists. Let Q denote the set of points $x \in g(A)$ at which there is a nonunique tangent; more precisely, we say that $x \in Q$ if there exist $w, w' \in A$ such that: (1) g is