MINIMAL SETS: AN INTRODUCTION TO TOPOLOGICAL DYNAMICS[1]

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The notion of minimal set is centrally located in topological dynamics. Topological dynamics may be defined as the study of transformation groups with respect to those properties, wholly or largely topological in nature, whose prototype occurred in classical dynamics.

Henri Poincaré was the first to introduce topological notions and methods in dynamics, that is, the study of ordinary differential equations. G. D. Birkhoff was the first to undertake the systematic development of topological dynamics, indicating its essentially abstract character and making fundamental contributions. Birkhoff's first paper on the subject appeared in 1912 [6, pp. 654–672]; Birkhoff's paper contains the first definition of minimal sets, some theorems about them, and some examples of them. Most of Birkhoff's work in topological dynamics from the point of view of general theory is to be found in Chapter 7 of his Colloquium volume *Dynamical systems* published in 1927 [5]. The Colloquium volume *Analytical topology* by G. T. Whyburn [27], published in 1942, contains related developments in its Chapter 12. The Russian book, *Qualitative theory of differential equations* by Nemyckiĭ and Stepanov [22], first edition in 1947 and second edition in 1949, contains a chapter devoted to topological dynamics; an English translation of this book has recently been announced by Princeton Press. The treatments of topological dynamics in the above books are all from the points of view of a single transformation or a one-parameter group of transformations. The Colloquium volume *Topological dynamics* by Hedlund and myself [14], published in 1955, is concerned mainly with general transformation groups.

A *topological transformation group*, or *transformation group* for short, is defined to be an ordered triple (X, T, π) such that the following axioms are satisfied:

(A0) (STIPULATIVE AXIOM). X is a topological space, called *the phase space*; T is a topological group, called the *phase group*; and π is a map of the cartesian product $X \times T$ into X, called the *phase map*.

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